Solving the Whistler-Blackcomb Mega Day Challenge

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The Whistler-Blackcomb (WB) Mega Day challenge requires a skier to ride all 24 lifts at the resort in a single day. Of the over two million people who ski annually at WB, only 313 completed the challenge in the 14 months following the introduction of a system that tracks lift use by skier. Apart from the physical challenge of skiing, one difficulty is finding a route that matches one's skill level while accounting for variable lift opening and closing times.

We use data from WB's radio-frequency identification (RFID) ticketing system to estimate ski times between lifts for skiers of various skill levels. We then formulate and solve the problem by a combined, iterative integer programming and heuristic approach, up to the highest feasible skier skill level. The problem's distinctive features preclude the use of known solution methods for similar problems; therefore, we use a practical, staged-solution approach.

Our results include a recommended route that enables the greatest number of skiers, roughly the quartile with the fastest skiers, to achieve the challenge. We also provide a benchmark, which skiers who can ski a particular common run in 12 minutes or less should be able to meet to complete the challenge. In the three months following communication of our recommended solution, the rate at which skiers completed the Mega Day challenge increased by two-thirds over the previous seven skiing months.

Keywords: skiing; sports analytic; routing problems; integer programming

History: This paper has been refereed.

Introduction

Whistler-Blackcomb (WB) ski area, located in British Columbia, Canada, hosted the 2010 Winter Olympics alpine events and is one of North America's largest ski resorts. WB spans more than 12 square miles across two mountains (Whistler and Blackcomb), each with more than 4,500 vertical feet of lift ski access.

In 2015, WB implemented a radio-frequency identification (RFID) ticketing system that enabled the resort to track movements of skiers, frequently numbering more than 15,000 per day, as they board 24 lift systems via 27 access points scattered throughout the resort. This RFID system offers management benefits, such as increased revenues through reduced ticket fraud, increased speed of service, and decreased staff costs. It also provides WB skiers with access to a Web-based portal, WB+, where they can view their personal statistics (e.g., number of rides and vertical metres per foot accumulated).

Whistler-Blackcomb was acquired by Vail Ski Resorts in 2017. WB+ follows a similar system in North America, EpicMix©, which is operated across several properties of Vail Ski Resorts and has been described as the 'gamification' of skiing (Khan 2010, Sean 2013). Detailed data on individual skiers enables Whistler-Blackcomb and Vail Ski Resorts to implement a motivational program through which skiers can earn a variety of badges, which EpicMix calls pins, based on their performance. For example, the Mount Everest badge is earned by riding a number of lifts in a single day, such that the sum of their vertical rises exceeds 29,029 feet. Some badges are based on total elevation and do not require riding specific lifts, as in this example, whereas others (e.g., the Whistler Complete or Blackcomb Complete) badges are earned by riding all RFID-enabled lifts on one mountain or the other during a single day.

Among the most challenging is the Mega Day badge, which a skier earns by riding all 24 lift systems on both mountains (i.e., 12 on Whistler, 11 on Blackcomb, plus the 'Peak-2-Peak' lift that spans the adjoining valley) in a single day.

Data made available to us showed that only a small number (~0.1 percent) of skiers earned the Mega Day badge on any given day; therefore, we set out to help WB management to improve the marketability of the WB award program by highlighting the Mega Day challenge as the pinnacle of this program. An important part of this effort was to demonstrate that this accomplishment is not only for expert skiers, but can be earned by skiers of modest ability if they follow a route that suits their skill level.

Our aim was to find minimum-time routes for skiers of varying ability, with emphasis on finding a route that would enable the greatest number of skiers to earn a WB Mega Day badge, that is, the minimum-time route for the least capable skier among those realistically able to complete the challenge. There are generally multiple trails that a skier can take from lift to lift, and the trail that is best in terms of speed and navigability may differ among skiers. More advanced skiers may follow steep trails that are shorter and faster for them, but that are difficult and ultimately slower (effectively longer) for less capable skiers. The former have more route flexibility, because a less advanced skier's feasible route is always feasible to the more advanced skier (although it may not be the latter's shortest time route for that skier's ability and pace).

Our approach was to solve the problem starting with an advanced skier's ability, which we characterize as a 1st percentile skier and then solve for increasing skier percentiles or decreasing ability. At some percentile, which turned out to be the 28th on our scale, a skier can expect to complete the challenge only by following a specific route, because there is little or no spare time to do otherwise.

Our results show that the WB Mega Day badge challenge is achievable by the top quartile of typical skiers at Whistler-Blackcomb, although a much smaller numbers have done so to date. In the three months following electronic-newsletter communication of our recommended solution to skiers at WB, the rate at which Mega Day challenges were successfully completed increased 67 percent. Our findings also suggest that if a skier can ski a common run from Roundhouse Lodge to the Whistler Village base in 12 minutes or less, that skier is capable of completing the Mega Day challenge. Finally, we found that if WB were to keep the Fitzsimmons lift open an extra hour (even if only on weekends), it could increase the accessibility of the Mega Day badge to skiers with a wider range of skiing abilities.

In the next section, *The Mega Day Challenge*, we define the problem and point out the relationships between this Mega Day routing problem and other similar problems in the literature. We then provide a mathematical formulation of our model, relating its components to our staged-solution approach. We discuss the RFID scan data made available to us, and how we parameterized the model using these data. Finally, we present our computational results and discuss how WB management has used them.

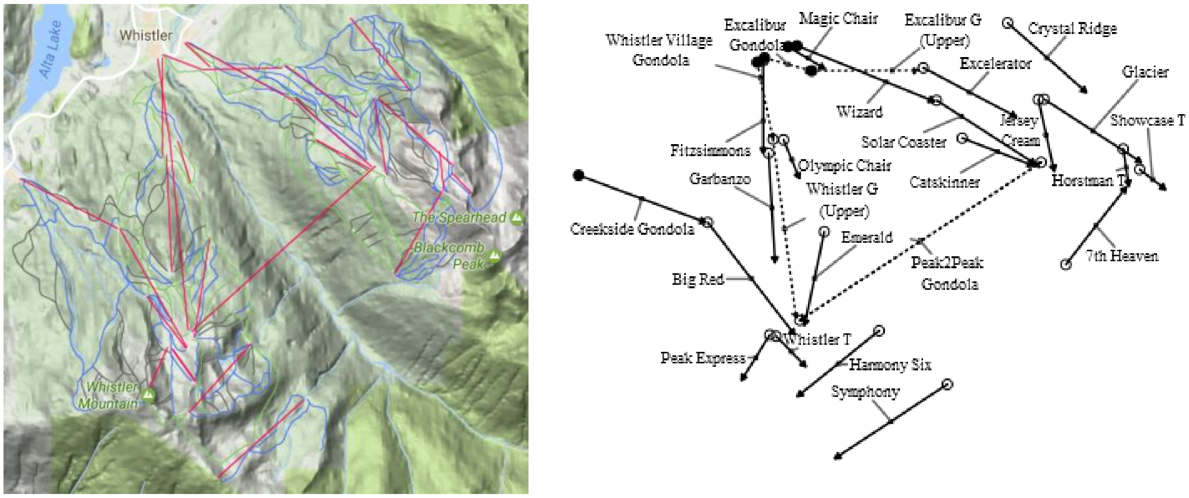
The Mega Day Challenge

A skier earns the Mega Day badge by having his (her) lift ticket scanned at each of the 24 RFID-enabled lift systems, which comprise the WB ski area, in a single day. Typically, scanners are located at the bottom entrance of the lift system; so, a route that directs a skier to visit the bottom of all 24 lifts would suffice. Although this seems like a straightforward routing problem, several features make this variant unique.

First, the resort includes three multisegment lifts; each is a lift system that includes more than one location at which a rider can get on or off. One of these lifts is the Peak-2-Peak Gondola, which spans the valley between Whistler and Blackcomb mountains and can be ridden in either direction. The other two are the Excalibur and Whistler Village gondolas; both are unidirectional but have mid-stations that can serve as alternate entry or exit points.

Figure 1 depicts a north-facing map of the Whistler-Blackcomb area on the left and a schematic of its 24 lift systems on the right. The dotted lines represent gondolas with multiple and (or) bidirectional segments. A skier must begin at one of seven potential starting points along the two mountain bases, indicated by solid black circles. These lifts are the Creekside Gondola, Whistler Village Gondola, Fitzsimmons Chair, Excalibur Gondola base, Magic Chair, Wizard Chair, and Excalibur Gondola mid-station (accessible from an upper-level parking area).

Figure 1. In this Map of Whistler-Blackcomb (Left) and Schematic View of the Ski Lifts (Right), Circles Represent Entry Points (Seven Possible Starting Lifts Shown as Solid Black Circles) and Dotted Lines Represent Lift Systems with Multiple Optional Segments



Source: www.googlemaps.com

We consider a route to be a sequence of lift segments ridden that meets the requirements of the Mega Day challenge. A route does not necessarily involve riding every segment of every lift and may include riding some lift segments more than once.

The seven possible starting lifts are spread over five kilometres, and open at slightly staggered times. Upper-level and back-valley lifts open progressively later in the morning, only after enough skiers can be in a position to reach these lifts. These remote lifts also close earlier in the afternoon, to shepherd skiers down toward the base of the resort, and allow ski patrols to complete sweeps before dusk.

A skier must check into his (her) final lift before it closes for the day. Neither the ride time on that final lift ride nor the following ski-out has any bearing on completion of the challenge.

In combination, these features make this routing problem unique and difficult to optimize. A key source of this difficulty is that we cannot impose hard time-window constraints on all lift segments, because we do not know in advance if they will be part of the chosen route. Also, the natural optimization objective of minimizing time gives preference to routes whose final lifts are most remote, because the subsequent, long ski-out time is not counted in the objective calculation. Conversely, early closing times of remote lifts make them poor choices as final lifts for slower (i.e., higher-percentile) skiers, the ones for whom solving the problem is most important. For these reasons, we have taken a practical approach that solves the problem in three stages.

The objective of our work was to create interest in the Mega Day challenge among skiers by providing recommendations and guidelines on the routes they should choose according to their abilities. In particular, we sought to identify routes that are feasible for the greatest number of skiers, from the most expert down to some skill level below which a skier cannot reasonably expect to be able to complete the challenge.

RFID Ticket System Information

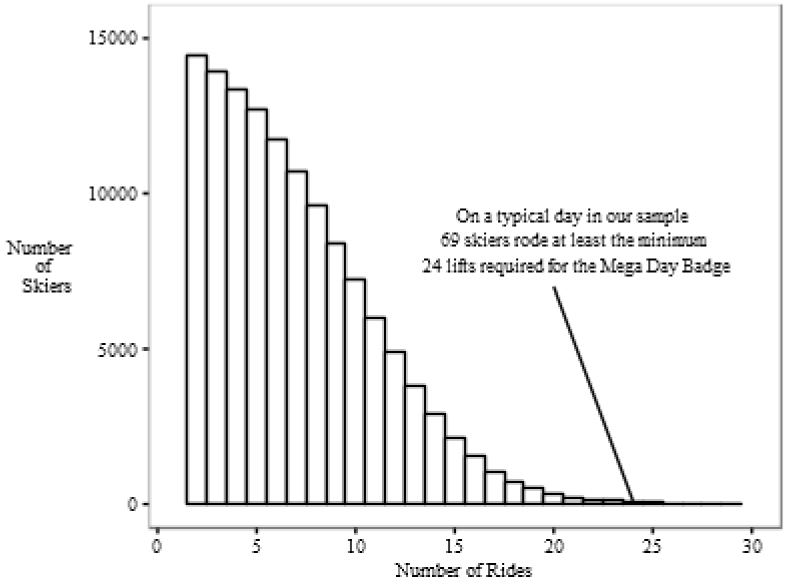
To develop and initially validate our model, we obtained a full day's worth of data for Thursday February 25, 2016 from the WB information technology (IT) business support team. We later obtained three additional days of RFID scan data to refine our model parameter estimates. Table 1 shows a summary of the scan data sets.

Table 1. We Obtained Summaries of Four Days of RFID Ski Lift Ticket Scan Data from WB+

Date (2016)	Feb 19 (Fri)	Feb 25 (Thu)	Mar 29 (Mon)	Apr 7 (Fri)
Unique RFID passes	19,591	15,417	17,152	11,665
Regular skiers	17,481	13,210	15,027	9,999
Total # scans	158,759	144,185	144,661	94,400
Regular-skier scans	145,823	128,689	130,609	83,641
Mega Day (MD) badges earned	1	14	16	11
MD routes with minimum rides (24)	0	1	4	1

Unique RFID passes represent the number of individuals riding at least one lift on each date. The regular-skiers category *excludes* RFID passes used for only one ride (mostly employees ascending to work at the upper-mountain lodges) and a small number of contractors and volunteers, such as law enforcement, who may have shared the RFID pass among skiers. The total # scans category represents all lift rides taken on each date, whereas regular-skier scans include only rides taken by regular skiers. The number of skiers who earned a Mega Day badge on each date is recorded. The final row in the table gives the number of distinct Mega Day routes taken on the date, which involved only the fewest possible number (24) of rides.

Figure 2. The Graph Shows the Distribution of Rides Per Skier on February 25, 2016



From our initial data set, we determined that among 13,210 regular skiers, only 69 (~0.5 percent) of them rode 24 or more lifts. Most of these 69 skiers did not earn a Mega Day badge, because they rode fewer than 24 distinct lifts; they rode multiple rides on some lifts and none on others.

We determined the existence of at least one Mega Day route involving only the minimum 24 rides by identifying three skiers who completed the Mega Day challenge together in this manner on February 25, 2016. The group members started at 9:42 am and checked into their final lift at 15:01 pm. This duration of 319 minutes is roughly three-quarters of the overall time available from first lift opening (8:15 am) to the resort's general closing time (15:45 pm), after which only one small base lift (Magic Chair) remains open, typically for an additional 60 to 90 minutes.

This evidence suggests that attaining the Mega Day badge is challenging; however, reasonably advanced skiers, and not exclusively expert skiers, should be able to achieve it, given that this three-person group achieved it on February 25 with more than two hours of open resort time to spare. A key question we sought to answer is how advanced a skier's ability must be for him (her) to have a reasonable chance of earning the Mega Day badge. The closer a skier's ability is to that threshold, the more important the skier's choice of which route to navigate through the mountain lift network will be. A feasible route for this marginal Mega Day skier will be feasible for any skier who is at a more advanced level.

Background Literature and Related Research

Routing problems have a long history in operations research. They extend back to 1736 when Leonhard Euler laid the foundations of graph theory, a commonly used technique to model such problems (Biggs et al. 1976). From the earliest days of electronic computing, the well-known traveling salesman problem (TSP) and its variants have served as benchmarks of combinatorial complexity and standards by which solution computational approaches, heuristics, and algorithms, are often compared (Cook 2012). In the early 1950s, a team from The Rand Corporation (Dantzig et al. 1954) developed an approach to large-scale instances of these problems, which would be described five decades later as "the Big Bang" that "all successful TSP solvers echo" (Jünger et al. 2010, p. 8). In their case, the specific problem was to find and prove a shortest-distance route passing through Washington, DC and 48 capitals of the lower-mainland U.S. states.

Over the years, many variations of the problem have been proposed and many more approaches have been employed to solve them. Entirely new classes of problems have developed and vehicle routing problems (Eksioglu et al. 2009) are among the broadest of them. We briefly identify a few related problem types that share critical characteristics with the Mega Day challenge.

The orienteering problem, also known as the selective traveling salesman problem (Vansteenwegen et al. 2011), has the objective of finding a route through a network of checkpoints, each of which has a specific score, where not all checkpoints must or can be visited within a given time frame. The Mega Day challenge is also selective, because it allows for only a subset of lift segments to be visited; however, its objective is binary (i.e., successful completion or not) and the rewards from visiting different lift segments have indistinguishable bearing on the decision of which lift segments to visit.

The lift network at Whistler-Blackcomb includes only 285 feasible transitions (excluding same-lift returns), of which 198 comprise 99 bidirectional connections. Only one pair of lift segments has both feasible connections and equal transition times in both directions. This makes our problem one of a broad class of asymmetric traveling salesman problems (Öncan et al. 2009).

Many routing problem variants involve time windows and (or) service times at the destinations (Kantor and Rosenwein 1992, Focacci et al. 2002, Campbell et al. 2011, Tas et al. 2016). Various approaches have been developed for the former, including using time buckets (Dash et al. 2012), adding variables for each destination's arrival time and constraining them to fall within that destination's time window (Desrosiers et al. 1995), and employing constraint logic programming (Pesant et al. 1998). However, these approaches were developed for problems in which all destinations must be visited, whereas with respect to the second of these approaches, for example, an unvisited lift segment has no arrival time in the Mega Day problem and so cannot be constrained to lie within a specified time interval. Ride versus ski times in the Mega Day problem could be seen as analogous to service versus travel times, because we consider entire lift segments as nodes, from entrance to exits with fixed ride times between them. However, bundling ride and ski times in our model allows us to reduce by half the number of nodes to define the problem.

Another related subclass of routing problems is the Steiner traveling salesman problem (Letchford et al. 2013) and its variants, including with time windows. Their important distinguishing features relevant to our problem are: (1) only a subset of nodes must be visited; (2) nodes may be visited more than once; and (3) edges between nodes may be traversed more than once. Any comparison to the Mega Day problem is deceptive, however. Rather than having required and optional nodes, we have groups of lift segments and require that one (or more) must be chosen from each group. A minimum time route could conceivably visit the same lift more than once, but is much less likely to be followed by the same subsequent lift.

Although there exists a large variety of closely related problem types and solution approaches in the literature, we have not found any among them that are like the Mega Day problem. Its uniqueness is due to the combination of the following features.

1. The network on which it is defined is clearly incomplete and highly asymmetric.

2. Route feasibility and time minimization depend not only on which lift transitions are chosen, but also the order in which lifts are visited.
3. The problem includes subsets of lift segments where only one segment needs to be visited (although more may be visited).
4. A subset of lifts that are possible starting points exists.
5. The lift-to-lift transition times are a combination of fixed lift ride times and variable ski times, the latter being a function of skier ability.
6. Like the Steiner TSP, the number of times that a lift may be visited is integer, not binary.

In addition to the related operations research literature discussed above, we identified a small body of research related to skier abilities and trail selections. Skier abilities have been characterized according to their linear velocities as measured by radar gun (Shealy et al. 2005). Our analysis of skier abilities is based on vertical speeds of descent. Graph-theory networks have been used to model the flow of skiers, particularly to analyze the cascading impacts of trail or lift closures on skier volumes and resulting queues (De Biagi et al. 2013). In that research, skiers have been segmented into three levels of ability, implying that they must choose between easy, intermediate, and difficult trails. Our research does not explicitly consider skier volumes or queueing, although we accommodate the latter in our time estimates (i.e., mean queue times are embedded within mean ski times). Our application of network and graph theory is to a route optimization problem for individual skiers rather than for modeling aggregate skier flow.

Methodology

We conducted our study in two phases. The first included data preparation and analysis, and provided input to the second phase in which we generated and solved instances of the problem for different skiers. The second phase utilized a three-stage solution procedure for each instance.

In the first phase, we cleaned and shaped RFID scan data from the WB+ system. We accounted for and removed exit scans to determine, per skier, what lift rides began at what times. We then gathered data about lifts, including lower and upper locations (i.e., latitude, longitude, altitude) and ride times. We translated pairs of successive rides into runs (i.e., transitions from lifts i to lifts j, where i and j are, respectively, the origin and destination in the transition, as we define in the appendix). We identified and removed infeasible observations (< 2 percent) stemming from scans that were occasionally missed between two lifts that have no direct interconnection. We disaggregated the ride and ski times, making adjustments in cases where multisegment lifts were exited at mid-stations. Finally, we derived $i \rightarrow j$ ski times according to skier abilities, from 1st to 100th percentiles, representing fastest to slowest skiers, respectively.

Our second phase was to develop and execute an optimization model. We derived a series of parameter data files from Phase 1 output for different skier abilities and executed the program in order of increasing skier ability. Feasible solutions to the Mega Day routing problem were obtained for skier abilities from the 1st up to and including the 28th percentile. These solutions were generated in the three-stage procedure we describe below. The first two stages are common to many integer programming approaches for solving routing problems:

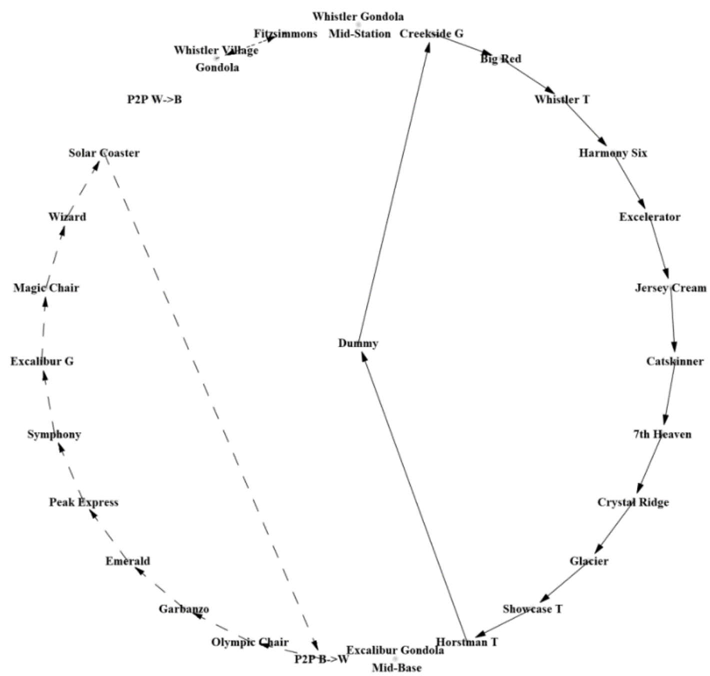
Stage 1 Solve the integer program (IP) described by Equations (1)–(5) in the appendix. Then determine whether the solution contains any subtours. If not, proceed to Stage 3 with the full-tour candidate solution.

Stage 2 Add subtour elimination constraints, Equation (6) in the appendix, as required, re-solving the IP and again determining whether the revised solution contains any new subtours. If so, repeat Stage 2.

Stage 3 Determine whether the full-tour candidate solution from Stage 2 satisfies time windows constraints, Equations (7) in the appendix. If necessary and possible, adjust start time and (or) accept delays imposed by arrivals in advance of lift opening times. If any lift closing times are violated, within the initial solution or because of time adjustments, return to Stage 2 with a new constraint, Equation (6), which precludes this full-tour candidate solution, and force instead a search for the next most optimal solution in terms of the Objective function (1).

Thus, a final solution for a skier percentile is first generated by Stage 1; then validated as physically feasible by Stage 2; and finally is validated for time feasibility in Stage 3, perhaps being adjusted with delays. We nevertheless refer to this as the 'shortest time route' for the skier level, because it is based on minimizing the objective function in Stage 1, regardless of time adjustments in Stage 3, if any. We visually show this in Figure 8 in the *Model Results* section.

Figure 3. An Example Shows Subtours Generated Within a Candidate Solution to IP Equations (1)–(5)



Optimal solutions to Equations (1)–(5) in Stage 1 may lead to the problem of subtours, for which we need to add instances of Equation (6) shown below. Figure 3 depicts a preliminary solution with three subtours, the smallest of which simply joins the Whistler Village gondola to the Fitzsimmons Chair and back again (shown just to the left of the image's top centre). Another subtour proceeds from the dummy lift to the Creekside gondola and eventually returns to the dummy lift after a visit to the Horstman T-bar. The underlying X_{ij} choices are optimal with respect to Equations (1)–(5); however, they do not form a meaningful route.

Table 2 provides an example of output from Stages 1 and 2 and its result in Stage 3 of our solution procedure. This is a minimum-time solution to Equations (1)–(5) in Stage 1 *plus* the set of subtour Constraints (6) needed to obtain a full tour in Stage 2. However, in Stage 3 we find that this candidate solution critically violates one or more time-window constraint Equations (7). Whereas the skier's expected early arrivals at the 2nd and 3rd lifts may be avoided simply by starting the tour later, late arrivals beginning at the 15th lift make this candidate solution infeasible with respect to time windows. Thus, we add another Constraint (6) to eliminate this full tour and seek the next best candidate solution to the updated set of Equations (1)–(6).

Table 2. This Example Shows an Ordered Set of Lifts that is a Solution to Equations (1)–(6) but Is Infeasible Because Of Time-Window Violations for Equations (7)

Order w	Destination Lift J(w)	Minutes Delay	HH:MM Depart	Minutes Ride & Ski Time	Stage 3 Result
1st	8120 Creekside G	-	8:15 am	11.41	
2nd	8130 Big Red	3.59	8:30 am	13.84	
3rd	8196 Whistler T	91.16	10:15 am	14.9	
4th	8180 Harmony Six	-	10:29 am	24.88	
5th	8210 Excelerator	-	10:53 am	10.16	
6th	235 7th Heaven	-	11:03 am	21.29	
7th	260 Catskinner	-	11:24 am	12.66	
8th	820 P2P B->W	-	11:36 am	27.28	
9th	145 Olympic Chair	-	12:03 pm	11.97	
10th	8140 Garbanzo	-	12:14 pm	15.38	
11th	8150 Emerald	-	12:29 pm	11.36	
12th	8190 Peak Express	-	12:40 pm	17.39	
13th	8195 Symphony	-	12:57 pm	32.33	
14th	8100 Whistler Village G	-	13:29 pm	14.54	
15th	8110 Fitzsimmons	-	13:43 pm	15.53	Lift Closed
16th	205 Excalibur G	-	13:58 pm	7.54	-
17th	234 Magic Chair	-	14:05 pm	9.07	-
18th	8200 Wizard	-	14:14 pm	12	-
19th	220 Solar Coaster	-	14:26 pm	19.89	-
20th	245 Crystal Ridge	-	14:45 pm	14.41	-
21st	8250 Jersey Cream	-	14:59 pm	13.86	-
22nd	240 Glacier	-	15:12 pm	10.59	Lift Closed
23rd	8165 Showcase T	-	15:22 pm	10.56	Lift Closed
24th	8255 Horstman T	-	15:32 pm	-	Lift Closed

Parameterizing the Model

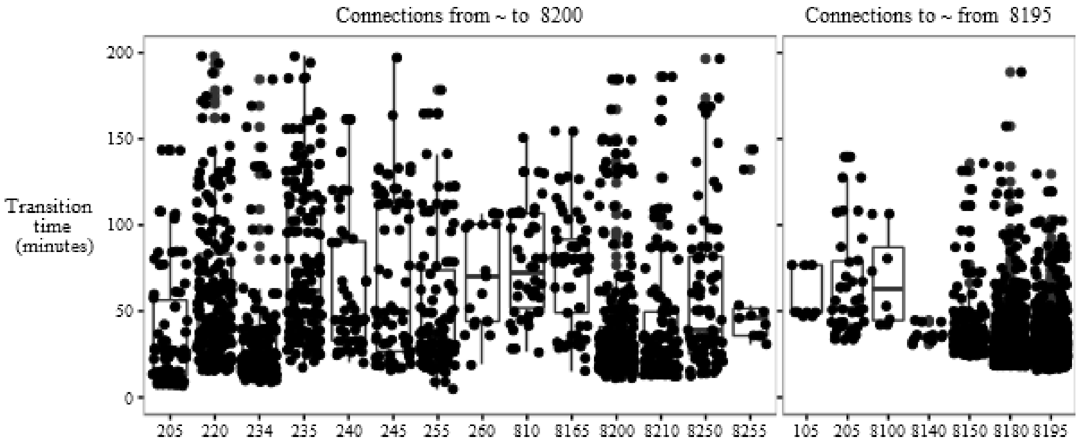
Table 3 provides a list of the $|N| = 27$ lift segments at WB, the elevations of their entry and exit locations, and their standard ride times T_i. To estimate transition times t_{ij} between lifts, we used time intervals between RFID scans collected at lift entrances. We first developed our methods using data from February 25, 2016, and then applied these data to four full days of scan data (Table 1).

Table 3. N Discrete Lift Segments Are Listed, Along With the Lift System G to Which Each Belongs, Base and Top Elevations (Metres above Sea Level), Standard Ride-Time Approximations, and Typical Opening and Closing Times

Lift Code	Lift Name	Lift System G	Base Elev (metres)	Top Elev (metres)	Ride Time (mins)	Lift Open (HH:MM)	Lift Close (HH:MM)
8100	Whistler Village Gondola	1	684	1,023	0:05:46	8:30 am	15:30 pm
105	Whistler G O Station	1	1,023	1,827	0:13:09	8:30 am	15:30 pm
145	Olympic Chair	2	1,020	1,145	0:06:30	8:45 am	15:45 pm
205	Excalibur Gondola	3	684	765	0:02:43	8:30 am	15:45 pm
255	Excalibur G Mid-Base	3	765	1133	0:05:36	8:30 am	15:45 pm
220	Solar Coaster	4	1,250	1,862	0:07:16	8:30 am	15:45 pm
234	Magic Chair	5	683	778	0:04:07	8:30 am	16:30 pm
235	7th Heaven	6	1,660	2,249	0:06:37	9:45 am	15:15 pm
240	Glacier	7	1,547	2,142	0:07:07	9:45 am	15:15 pm
245	Crystal Ridge	8	1,282	1,822	0:06:34	9:30 am	15:45 pm
260	Catskinner	9	1,539	1,860	0:09:10	10:00 am	15:45 pm
810	P2P W->B	10	1,825	1,878	0:11:07	9:15 am	15:45 pm
820	P2P B->W	10	1,878	1,825	0:11:07	9:00 am	15:45 pm
8110	Fitzsimmons	11	689	1021	0:06:43	8:30 am	11:00 am
8120	Creekside G	12	661	1302	0:07:52	8:15 am	15:30 pm
8130	Big Red	13	1,301	1,846	0:09:22	8:30 am	15:45 pm
8140	Garbanzo	14	1,021	1,676	0:07:43	8:30 am	15:45 pm
8150	Emerald	15	1,413	1,834	0:06:51	8:45 am	15:45 pm
8165	Showcase T	16	2,146	2,274	0:03:28	10:00 am	14:30 pm
8180	Harmony Six	17	1,584	2,102	0:06:07	9:15 am	15:00 pm
8190	Peak Express	18	1,771	2,172	0:03:46	9:15 am	15:15 pm
8195	Symphony	19	1,529	2,027	0:07:43	11:00 am	14:30 pm
8196	Whistler T	20	1,786	1,962	0:05:27	10:15 am	15:15 pm
8200	Wizard	21	688	1252	0:08:27	8:30 am	15:30 pm
8210	Excelerator	22	1,131	1,635	0:07:01	8:30 am	15:30 pm
8250	Jersey Cream	23	1,547	1,912	0:05:21	8:45 am	15:30 pm
8255	Horstman T	24	2,047	2,250	0:06:57	10:30 am	15:00 pm

After adjusting for exit scans and removing anomalies from some missing intermediate scans, we obtained sets of observed $i \to j$ transition times for various lift pairs. Figure 4 shows boxplots of example transition times observed on a given day. The set at the left are times recorded from 14 different origin lifts leading *to the* Wizard Chair 8200. On the right, we show times recorded from the Symphony Chair 8195 to seven destination lifts recorded on the day.

Figure 4. The Boxplots Show Transition Times from 14 Lifts to Wizard Chair 8200 (Left), and to 7 Lifts from the Symphony Chair 8195 (Right)



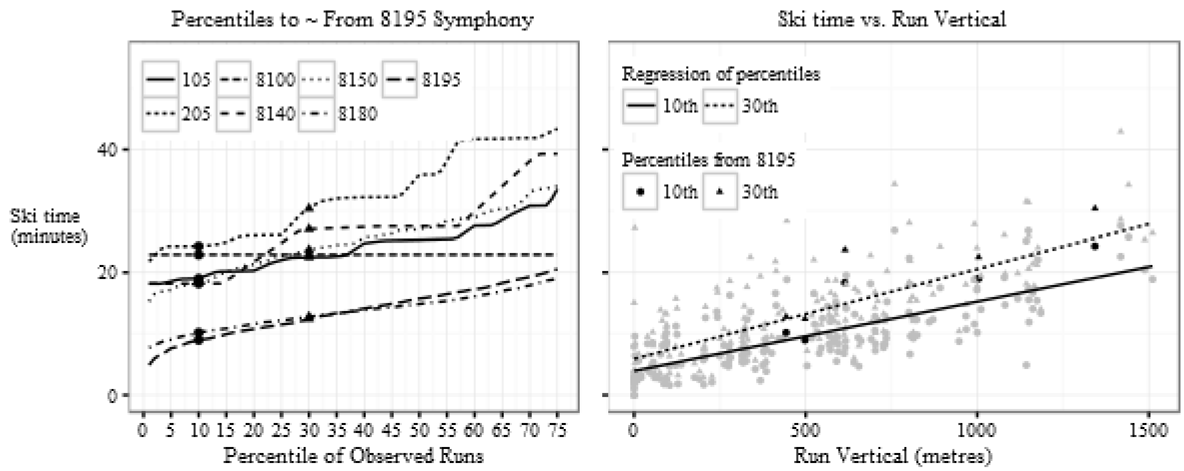
We eliminated observations of transition times greater than 60 minutes, on the premise that skiers likely stopped for refreshments at some point during those runs.

Our model did not explicitly consider lift queues and wait times, because we had no basis on which to disaggregate them from total transition times. RFID scans were captured only at points just prior to lift boarding, and provided no record of any preceding queue. Some extra time to reflect expected delays is embedded in each t_{ij}^k parameter estimate by our approach to their derivation, as we describe below.

Some feasible lift-to-lift transitions had no observations in our data sets. Therefore, two feasible destinations from Symphony Chair 8195 are missing from Figure 4. However, we see two destinations (including a return to lift 8195 itself) that had well over 2,000 observations per day to derive meaningful percentile ski times.

The left side of Figure 5 provides a different view of transitions from Symphony Chair 8195. It depicts percentile ski times (i.e., RFID scan-to-scan interval minus fixed ride time) from 8195 to its various recorded destination lifts. The 10th and 30th percentile observations are marked as circles and triangles, respectively. Five of these pairs are highlighted again in the right side of Figure 5, as we explain in the next paragraph.

Figure 5. On the Left, Observations of Lift-to-Lift Ski Times Are Parsed into Percentiles; on the Right, Two Linear Regression Models, Derived from Transitions of >=25 Observations, Provide Estimated Skier Percentile Ski Times as Functions of Run Vertical Distance

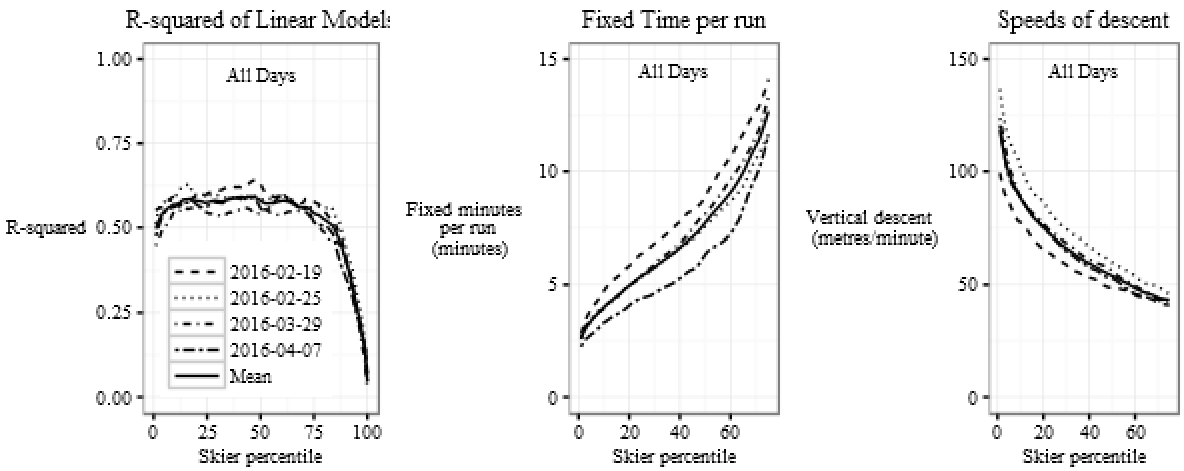


We used percentiles data only from transitions having at least 25 observations (180 out of 312 feasible transitions at WB, including same lift returns). We formed 100 sets of data by percentiles, each set with coordinate pairs $\{h_{ij} \equiv \text{vertical\_distance}(i, j), y_{ij}^k \equiv \text{percentile\_k\_ski\_time}(i, j)\}$ to which we fit linear models of the form $\hat{y}^k \sim \alpha^k + \beta^k \mathbf{h}$. These linear models were used to parameterize ski times $\hat{y}_{ij}^k = \alpha^k + \beta^k h_{ij}^k \cong \tau_{ij}^k$ for each percentile skier to ski each feasible lift transition, regardless of whether and how many of those transitions were observed in the data set.

The right side of Figure 5 shows two of these linear models, for the 10th and 30th percentiles. The points depicted as dark circles and triangles correspond to five pairs of data on the left, that is, the 10th and 30th percentile times for like transitions from 8195, among transitions with 25 or more observations only.

The higher-quantile linear model has both a steeper slope and a higher intercept. The latter point is consistent with the difference in ski time between skiers of different abilities being amplified by the length of a run. The former accounts for skiers of different abilities also being slower (faster) to navigate cordoned lift lineups, and (or) to prepare themselves to initiate a run after disembarking. This fixed-time component may also account for stops on trails that are made more frequently by less advanced skiers.

Figure 6. We Compare Linear Regression Models (Ski Time ~ α + β * metres) Across Skier Percentiles and Days, Including R^2 Values (left), Model Intercepts (α) Representing Fixed Time Components (Centre), and Speeds of Descent (Right)



Our linear models had consistent R-squared values in the range of 0.5-0.6, across all percentiles below the 75th. Intercept estimates increased progressively from a base of three minutes for the lowest percentile, fastest skiers. We consider these fixed times in our model as buffers to account for navigation around lift entries and exits, regardless of actual ski time between lifts. The inverses of our linear model slope parameters are shown in the right side of Figure 6, that is, to depict metres per minute (rather than minutes per metre). We do this to simplify characterization for a 28th percentile skier, the highest for which we found a solution to the Mega Day problem, as one whose typical rate of descent we estimate to be roughly 65 metres (200 feet) per minute.

We used the four-day mean values of our linear model parameters to calculate τ_{ij}^k for all feasible runs at each successive skier percentile k. These calculated ski times, added to the fixed ride time of the origin lift in each case, serve as the objective function coefficients $t_{ij}^k = T_i + \tau_{ij}^k$ for each execution of our model for $k = 1, 2, ...$

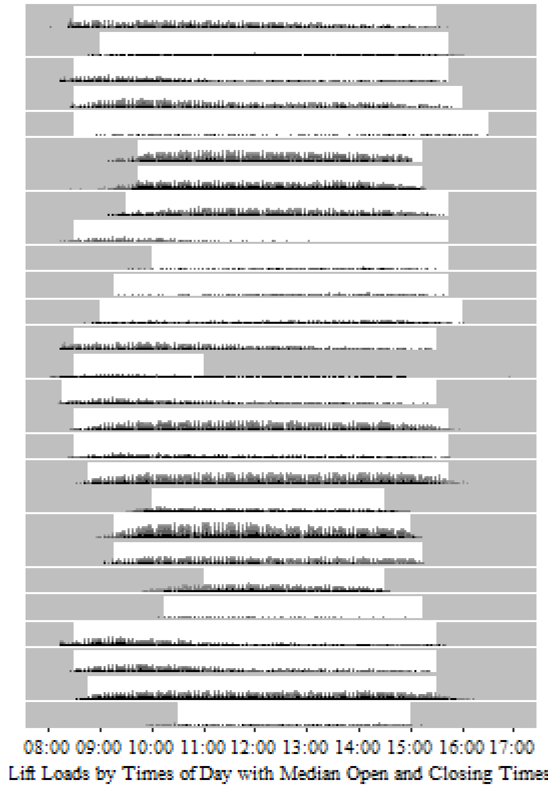
Time Windows

We derived lift time windows empirically from our RFID scan data. Figure 7 depicts the relative volumes of skiers riding each lift, per minute-of-day interval, summed over four days of our sample data. (Refer to Lift

Codes in Table 3 for lift names and other details). Lift opening and closing times often vary from day to day; therefore, for each lift, we used the mean of its four-day observations rounded to the nearest quarter-hour for our model lift window opening times and lift closing times.

The fifth row of Figure 7 shows that Magic Chair (234) remains open well after 15:45 pm by which time all other lifts are typically closed. The 14th row shows that Fitzsimmons Chair (8110) is generally open for only the first few hours of the day (although told us that it is open longer on busy holidays). The Whistler T-bar (8195), which lies in an upper section of Whistler Mountain and provides access to the back-valley side, generally opens late and closes early (e.g., 11:30 am-14:30 pm on February 25, 2016). Although inclement weather, mechanical maintenance, and other factors occasionally require adjustments to lift opening and (or) closing times, we treat time windows as deterministic in our model.

Figure 7. We Show the Volumes of Skiers Riding Each Lift, per Five-Minute Interval over Four Days

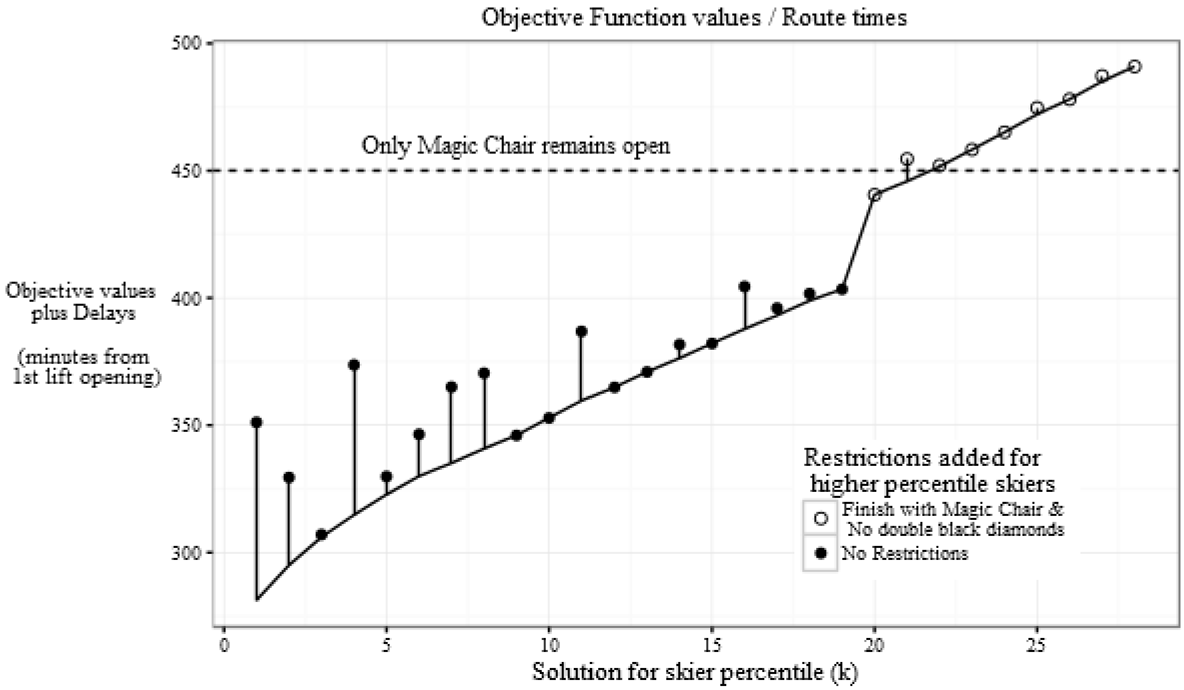


Note. Typical lift time windows are represented by white sections in each line.

Model Results

We executed our model with parameters derived for skier abilities from the 1st to 30th percentiles (unsuccessfully for the 29th and 30th percentiles). In several instances, expected completion time exceeded the objective function value by a sum of delays incurred to adjust the route in Stage 3, because of early expected arrivals at some lifts. In most cases, delays could be merged into a late start, such that the route would still lead to the shortest elapsed time, from start to finish, for the given skier percentile.

Figure 8. Solutions for the 1st to 28th Percentile Skiers Are Shown with Increasing Objective Function Values Represented by the Continuous Line



Note. Circles depict expected completion times after delays for time windows imposed by Stage 3.

Using the IBM ILOG CPLEX 12.3 MIP solver on an Intel i5 1.8 GHz processor, solutions were generally obtained in under two minutes, up until the 20th percentile skier where we did not initially obtain a solution through several hours and more than 3,000 iterations of Stages 1–3. We noted at that point that the objective function value (i.e., expected finish time without contingency for delays) had exceeded the closing times of several favored final-destination lifts in previous lower-percentile solutions. Moreover, those earlier solutions often took advantage of three short but very steep transitions, called double black diamond runs.

We thereafter specified that routes for higher-percentile skiers should end with the Magic Chair to take advantage of its much later closing time. Simultaneously, we assumed that less advanced skiers would likely prefer not to ski any of the double black diamond runs; therefore, we removed those from our set of feasible $i \rightarrow j$ connections. With these changes, we obtained rapid solutions up to the 28th percentile, albeit with elevated objective function values and expected finish times.

The significant vertical gap in Figure 8 between the finish times of the 19th and 20th percentile skiers stems from the latter being unable to reach a final lift near the top of the mountain on time, following a route that includes Magic Chair at an earlier stage. However, leaving it as their final lift (i.e., all 20th percentile and above skiers finish with Magic Chair) implies that the clock continues to run during a long final transition down to Magic Chair at the base of Blackcomb Mountain. This long transition also starts progressively later and takes progressively longer with each increasing skier percentile.

Aside from suggesting these additional restrictions for higher-percentile skiers, our analysis of interim candidate solutions (i.e., from Stages 1 and 2) provided other interesting insights. First, we noted that many early candidate solutions incurred time violations at the Fitzsimmons lift between its regular 11:00 am closing time and noon. The implication is that if WB were to keep the Fitzsimmons lift open an extra hour (even if only on weekends), it could vastly increase the accessibility of the Mega Day badge to include a wider range of skiing abilities. Second, although all final solutions included only 24 lift segments, Stages 1–2 occasionally produced candidate solutions that included multiple rides on a single lift. However, none of these turned out to be time-feasible full-tour solutions.

Solutions for many percentiles shared many common lift subsequences; yet, we obtained 27 distinct routes among 28 different percentile solutions. We explain this result by the fact that transition times increase disproportionately for different skier percentiles. Run b may be twice as long as run a for a 20th percentile skier, but merely one-and-a-half times as long for a 10th percentile skier; consequently, run b and (or) its successors may trigger a time-window violation for the slower skier, but not for the faster skier.

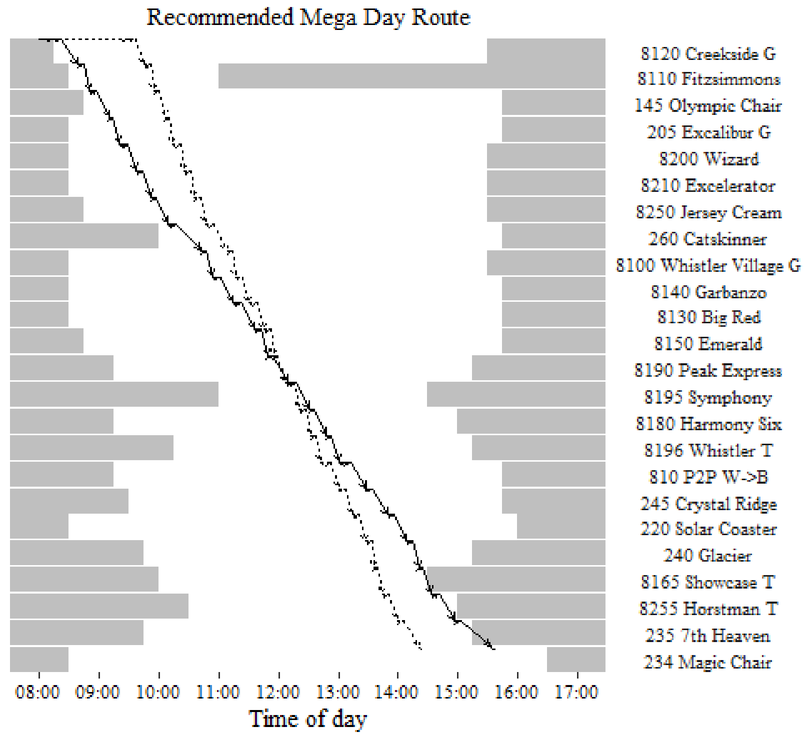
Recommended Route and Implementation

The Mega Day challenge has considerable marketing appeal; however, a skier who is looking at a trail map and trying to determine the best way to traverse through all 24 lifts faces the prospect of committing to a difficult ski day and then perhaps being disappointed because a lift or the resort closes before he (she) can complete the intended tour.

We proposed the recommended Mega Day challenge route based on the solution with the shortest time and feasible time windows for the 28th percentile skier. This route is depicted in Figure 9, with vertical axes showing lift rides in sequence, including time windows. We also proposed a recommendation to skiers at Whistler-Blackcomb, based conservatively on the pace of a 20th percentile skier, that is, “if you can ski from the Roundhouse lodge to the Whistler Village base in under 12 minutes, you should be quite capable of completing the Mega Day challenge.”

We communicated our solution and recommendations to the WB management team and delivered them to skiers through an electronic newsletter. Although WB Communications noted that this content received unusually high website user ‘engagement” as measured by web page browsing time, determining the direct impact is difficult. Statistics reveal, however, that in the three months following the newsletter’s publication, 224 Mega Day badges were achieved, compared with 313 over seven prior skiing months (a 67 percent increase). Following the acquisition of Whistler-Blackcomb by Vail Ski Resorts during the 2016-2017 season, integration of WB+ into the Vail Ski Resorts EpicMix® system will expose a much larger community to the Mega Day challenge and presumably motivate even greater numbers of skiers to undertake and complete the challenge.

Figure 9. The Solid Line Shows the Recommended Mega Day Route with Expected Timing for a 28th Percentile Skier; the Dotted Line Shows the Same Route with Timing at a 1st Percentile Skier's Pace and with a Delayed Start



The Mega Day route we recommend for a 28th percentile skier (depicted in Figure 9) should have the greatest possibility of being completed by the largest number of skiers, because faster skiers can always delay their start time and (or) adjust their pace of skiing to follow this route.

Validating Our Proposed Route

Author Lyons skied WB on January 22, 2017 and set out to follow a route recommended by our model for a 10th quantile skier. He boarded the first lift at 8:42 am and, notwithstanding a detour caused by temporary closure of one lift, completed the Mega Day challenge at 15:20 pm (398 minutes). This time is quite consistent with times suggested by the model. Data obtained later showed that Lyons was the lone skier to complete a Mega Day challenge among over 15,000 skiers on the mountains that day.

Subsequently, we received an additional data file containing records of all Mega Day challenge awards completed from the inception of the RFID-WB+ system in December 2015 up to late January 2017. These data showed that 313 Mega Day badges were earned by 295 distinct skiers up to that point. In 138 of those cases, the Mega Day skier rode only the minimal number of 24 lifts. In many cases, the challenge was completed by groups of two to four skiers skiing together; up to that date, 64 Mega Day expeditions had been successfully completed.

We determined the routes followed in these 64 instances. These groups started from six different points: Whistler Village Gondola (19), Fitzsimmons Chair (31), Excalibur Gondola (2), Wizard Chair (1), Creekside Gondola (7), and Magic Chair (4). A surprising result was that each skier-group route was distinct: no two groups followed the same sequence of 24 lifts!

These findings suggest that the Mega Day challenge is a difficult accomplishment partly because there is no recognized or obvious 'optimal' route, although many routes are possible for strong skiers. Weaker skiers need to select their routes more carefully if they are to accomplish the feat before the last lift closes.

Conclusion

WB's introduction of the RFID system and associated WB+ web application make extensive skier data available to enhance the skier's experience by providing challenges and offering rewards. This 'gamification' of skiing opens up opportunities in the realm of analytics that mirror the rapid growth in the application of analytics in other predominantly team-based sports over the last two decades.

We undertook this study initially because we were intrigued by the easily relatable, real-world nature of the problem, and its relationship to a variety of routing problems addressed by operations research. We were further motivated by discovering how few skiers accomplish this feat, partly we believe, because of the difficulty in imagining a feasible route for their skiing ability. In the research process, we were able to develop a reasonably efficient model and to parameterize the model with real data to produce results consistent with actual records of successful Mega Day challengers. The guidance that we have been able to provide should not only inspire more skiers to attempt the feat, but also give them an appreciation for the type of operations research problem-solving approach on which it is based.

Appendix

Model Formulation

Our mathematical formulation is presented in three stages that coincide with the solution procedure.

Integer Program (Stage 1)

Twenty-seven discrete ski lift segments are each mapped to one of 24 lift systems. The former are required to specify which lift-to-lift connection points are chosen, and to calculate the objective function value associated with those choices, whether they comprise a full tour or multiple subtours. The mapping is required to verify whether all lift systems are represented among the origins and (or) destinations of the chosen connections. We define the following sets, parameters, and variables:

N A set of discrete lift segments (generally referred to simply as lifts).

G A set of lift systems (groupings of one or more lift segments).

B A subset of lift segments $B \subseteq N$ that constitute feasible starting lifts.

g_m A subset of lift segments belonging to the same lift system $g_m \subset N, m \in G$.

T_i The standard ride time (fixed) from the base to the top of a lift segment $i \in N$.

We note that lifts may operate at various speeds, depending on load and other factors. However, we assume, for simplicity, fixed lift ride times, as suggested by Whistler-Blackcomb personnel.

τ_{ij}^k The ski time from the top of lift *i* to the base of lift *j* for a skier of skill level *k*.

t_{ij}^k The total transit time from the base of lift *i* to the base of lift *j* for a skier of skill level *k*.

Note that $t_{ij}^k = T_i + \tau_{ij}^k$.

For contiguous lift segments i, j belonging to the same lift system g_m, we specify the ski time $\tau_{ij}^k = 0$, showing that a skier merely needs to remain onboard to ride the second segment.

Recognizing that transition times are generally specific to the skier's skill level, we drop the skill level superscript k from this point forward.

We define:

$\hat{N}$ The union of the set N with a "dummy lift" $i = 0$, that is, $\hat{N} = N \cup \{0\}$.

t_{0j} The transition time assigned to a skier for travel from the initial dummy lift to the entrance of any possible starting lift in B. We assign $t_{0j} = 0$, " j Î B, $t_{0j} = \infty$ otherwise.

t_{i0} The transition time assigned to a skier returning from lift *i* to the dummy, $t_{i0} = 0, \ \forall \ i$.

X_{ij} The decision variables, where $X_{ij} = 1$ if the route for a skier includes skiing (or connecting within the same lift system) from top of *i* to bottom of *j*, otherwise $X_{ij} = 0$. We also set $X_{ij} = 0$ for infeasible $i \to j$ transitions.

S_j the number of times a skier visits (rides) a lift segment j, that is, $S_j = \sum_{i \in \hat{N}} X_{ij}$.

The objective is to find the set of decision variables X_{ij} that minimizes the time to visit all lift systems at least once:

$$\textit{Minimize} \quad \sum_{i \in \hat{N}} \sum_{j \in \hat{N}} X_{ij} t_{ij} \tag{1}$$

subject to

$$\sum_{j \in g_m} S_j = \sum_{j \in g_m} \sum_{i \in \hat{N}} X_{ij} \geq 1 \qquad \forall \ m \in G \tag{2}$$

$$\sum_{i \in \hat{N}} X_{ik} = \sum_{j \in \hat{N}} X_{kj} \qquad \forall \ k \in \hat{N} \tag{3}$$

$$\sum_{j \in N} X_{0j} = \sum_{i \in N} X_{i0} = 1 \tag{4}$$

$$X_{ij} \text{ binary}, \quad S_j \text{ integer} \tag{5}$$

All components of the Objective function (1) that involve the dummy lift (either $i = 0$ or $j = 0$) evaluate to zero. Consequently, neither a skier's final ride nor his (her) final ski run add any cost (time) to the objective function value.

Constraints (2) allow solutions to use a given lift segment more than once, but specify that, for all lift systems g_m, the sum of the visits to its member lift segments $j \in g_m$ must be at least one.

Constraints (3) are for flow conservation. The number of arrivals that flow into any lift in $\hat{N}$, including the dummy, must be the same as the number of departures flowing from that lift.

Constraint (4) ensures that there is only one connection from the dummy lift 0 to (one of seven practical starting lifts in) the real lift network N, as well as only one connection from the real lift network back to the dummy lift.

Constraints (5) specify that the choice of whether a solution includes travel from i to j is binary, whereas the number of times each lift system is ridden, along any of its segments, must be an integer equal to or greater than one.

Subtour Elimination (Stage 2)

Following the DFJ formulation named for three integer programming pioneers who introduced it (Dantzig et al. 1954), we use 'lazy' constraints to eliminate subtours. We first define:

Z A set of edges (lift-to-lift transitions) comprising a subtour that we wish to eliminate.

We then add constraints:

$$\sum_{z \in Z} X_{I(z)J(z)} \leq |Z| - 1, \quad \forall\, Z \tag{6}$$

These constraints ensure that at least one of the connections in the subtour cannot exist if all the others do, that is, to require a solution that has at least one connection into and one connection out of the subset Z, respectively from and to the complementary set of lifts N\Z.

Timeline Validation, Route Adjustment, and (or) Rejection (Stage 3)

After obtaining a full-tour candidate solution to Equations (1)–(6) as output from Stage 2, we check to see if the solution satisfies the time-window restrictions of each successive lift in the tour.

We define the following sets, functions and variables:

W An ordered set of edges $i \rightarrow j$ chosen within a full-tour candidate solution, where we denote the l^{th} element of the set as W_l.

$I(w)$ A function that returns the origin lift $i \in \hat{N}$ from an edge $w \in W$.

$J(w)$ A function that returns the destination lift $j \in \hat{N}$ from an edge $w \in W$.

$T^w_{I(w)}$ The time at which a skier is expected to depart from an origin lift *I(w).*

$T^w_{J(w)}$ The time at which a skier is expected to arrive at a destination lift *J(w).*

$O^w_{I(w)}$ The opening time for an origin lift $I(w)$ of an edge w.

$O^w_{J(w)}$ The opening time for a destination lift $J(w)$ of an edge w.

$C^w_{J(w)}$ The closing time for a destination lift $J(w)$.

Noting that $T^w_{J(w)} = T^w_{I(w)} + t_{I(w)J(w)}$ and $O^w_{J(w-1)} = O^w_{I(w)}$, we calculate the starting time of Stage w, which begins at lift $I(w) = J(w-1)$, as follows:

$$T^w_{I(w)} = \begin{Bmatrix} O^w_{J(w)} & \text{for } w = W_1 \\ Max\left\{T^{w-1}_{J(w-1)}, O^w_{I(w)}\right\} & \text{for } w = W_l \ \ (\forall l \neq 1) \end{Bmatrix}$$

The starting time for the first stage is the opening time for the first destination lift (departing from the dummy lift). For the remaining stages, the starting time is the greater of the arrival time from the previous stage, and the opening time of the origin lift in the current stage. The difference, if any, represents an amount by which the start of leg w is delayed. We add the constraints:

$$T^w_{J(w)} \leq C_{J(w)}, \qquad \forall \ w \in W \tag{7}$$

Constraints (7) specify that each ordered transition w must be completed before the closing time of its destination lift.

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