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Using a spark-spread valuation to investigate the impact of corn-gasoline correlation on ethanol plant valuation

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ABSTRACT

Corn ethanol plants have been criticized for a number of reasons in recent years. This paper provides another ground for criticizing these plants. Historical corn and gasoline prices are uncorrelated, but widespread adoption of corn ethanol production might reasonably lead to future correlation between these prices. We present a real options — like valuation of an ethanol plant as a spark spread between the corn price and the gasoline price. This analysis shows that the value of an ethanol plant monotonically decreases with increasing correlation and the optimal production schedule greatly depends on the correlation. Even relatively small new correlations can result in a significant proportional value decrease; a 50% correlation between corn and gasoline causes ethanol plants to lose 10% of their value. The limiting case of full correlation would lead to a 30% value loss.

1. Introduction

In recent years it has become both politically and, to a lesser extent, environmentally popular to produce fuel ethanol from corn (this is different from sugar cane ethanol which is used in Brazil). This process has also received some criticism. One criticism that has been advanced is that the process is energy negative — that more oil is used to grow the corn and convert it into ethanol than is produced in the process (Kim and Dale, 2005; Patzek et al., 2005; Pimentel, 2003; Shapouri et al., 2002). We do not enter that particular debate here except insofar as to note that corn ethanol production must receive government subsidies to be economically attractive (Koplow, 2006). Another criticism is that converting corn production from food or feed use to fuel use is responsible for an increase in food prices with concomitant ill effects for the world's poor (Pimentel and Patzek, 2005).

In this paper we do not address the relationship between corn ethanol production and food prices directly, but rather consider the possibility that widespread ethanol production will cause the price of gasoline and the price of corn, which historically have been nearly uncorrelated, to become more highly correlated in the future. We present calculations which suggest that possible future correlations between corn and gas prices may impact the level of public subsidy required in order to prevent the value of ethanol plants from being destroyed. To show this, we model an ethanol plant as a real option on the appropriate weighted spread between gasoline (output) and corn (input) prices. We use the real options framework (see Wilmott, 2000) because recent historical experience suggests that ethanol plants may not always be economically advantageous to operate even with a subsidy; however in this case the plant itself still retains value because of the ability it represents to profitably run in the future. The resulting model may be computed using a variety of techniques including a bootstrap-based simulation. With the aid of this model we are able to quantify the value destroyed, for a realistic but simplified model ethanol plant, by a given level of future corn-gasoline correlation. We draw public policy conclusions from this result.

Section 2 provides insight from a simple Margrabe option valuation to show that ethanol plant value loss is inevitable. Section 3 introduces the idea of a spark spread option and derives the formula for valuing an ethanol plant using this approach. Historical spot price data is used in applying the formulation in Section 4. In Section 5 we explore corn and gasoline price dynamics in order to generate price sequences for both commodities using a bootstrapping technique in Section 6. These price

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sequences are generated with a specified level of correlation between the commodity prices, and the effects of increasing correlation are reported. A more thorough discussion of the results is provided in Section 7.

2. A simple Margrabe options lesson

A Margrabe exchange option is a contract which gives the owner the right, but not the obligation, to exchange b units of an asset S_1 for a units of an asset S_2 at maturity T (Margrabe, 1978). The payoff is therefore

$$(aS_1(T) - bS_2(T))^+. \tag{1}$$

Since we want to continuously make the decision of whether or not to convert corn to ethanol, we can consider our problem a strip of exchange options, each having a payoff similar to Eq. (1). The advantage of this consideration is that there is a formula for valuing exchange options; the formula is given below.

$$M = S_1 e^{(\mu_1 - r)\tau} N(d_1) - S_2 e^{(\mu_2 - r)\tau} N(d_2) \tag{2}$$

where

$$d_1 = \frac{\ln(S_1 / S_2) + (\mu_1 - \mu_2 + \sigma^2 / 2)\tau}{\sigma\sqrt{\tau}},$$

$$d_2 = d_1 - \sigma\sqrt{\tau},$$

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}$$

and τ is the time until expiry.

We use this formula to build intuition regarding the effect of varying the correlation ρ: it is clear from the above equation for σ that as ρ increases, σ decreases. If we look at the corresponding effect on Eq. (2) it is easy to see that as ρ increases, M decreases. This is shown in Fig. 1 for an initial fixed corn and gasoline price pair.

Since in a Margrabe-type option the exchange happens just once, at time T, this valuation is not sufficient for our problem. At each time t we wish to examine the difference between the price of corn and gasoline to make a decision. Thus, we rely on other existing tools to value a corn ethanol plant which allow us to continuously make decisions.

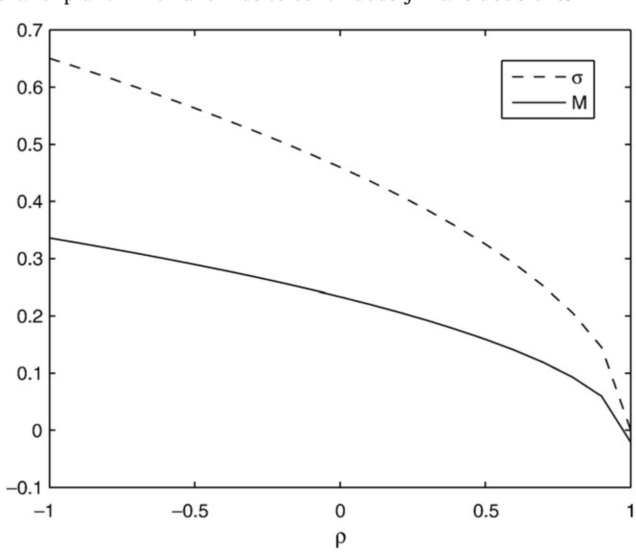

Fig. 1. The Margrabe parameter σ as ρ increases (dashed line), and the value of the option (M) as ρ increases (solid line). Parameter values: $\sigma_1 = 0.3252$, $\sigma_2 = 0.3252$, $\mu_1 = -0.0265$, $\mu_2 = 0.0125$, $r = 0.04$, and $S_1(0) = S_2(0) = 2$.

3. Spark spread options

Spread options are extremely valuable decision making tools in situations where the difference between the prices of two assets determines the economic value of an operation. Spark spread options are used in electricity generation where the spread between the price of fuel input and electricity output is examined in order to decide whether or not to run a particular generation asset (Deng et al., 1994).

Ethanol producers can use spark spreads in a similar way. They can examine the appropriate spread between corn (input) and gasoline (output) to determine whether or not they will be profitable at a given time. If the spread becomes negative, they have a decision to make: should the plant run at a loss, or should the plant temporarily shut down and resume production when the spread becomes positive again? If the latter approach is taken, the situation is similar to that of an option where the owner of a contract *chooses* whether or not to exercise their right to buy or sell at a given time. We therefore value the operation as a call–like option where, at each time t, the payoff is

$$\pi_t = (HG_t - C_t)^+. \tag{3}$$

Here, π is the profit per unit of gasoline, G_t is the spot price of gasoline (dollars/gallon), C_t is the spot price of corn (dollars/bushel) and H is the conversion factor from gasoline to corn units (gallons/bushel). Thus, the plant is run when

$$H \times \text{gasoline price} > \text{corn price},$$

otherwise it is unprofitable to do so and production should cease. This valuation method differs greatly from the method of discounted cash flows, which is likely to undervalue the operation since it does not capture its flexibility. Unlike a call option, both G_t and C_t are unknown for $t>0$. We must compute a suitable range of H values to use in our simulations. A range of values exist since the energy content of gasoline and corn is not constant. For instance, gasoline blends may differ with batch, season and from one refinery to another. The energy content of one bushel of shelled corn is given by $\beta = 448,000 - 476,000$ British thermal units (Btus) while the energy content of one gallon of gasoline is given by $\gamma = 115,000 - 125,000$ Btus. One bushel of corn gives approximately $\alpha = 0.8$ bushels of shelled corn. Thus, we can compute H to be in the range

$$\begin{aligned} H &= \frac{\alpha\beta}{\gamma} \\ &\approx 0.8 \times \left(\frac{448,000}{125,000}, \frac{476,000}{115,000}\right) \\ &\approx 2.87 - 3.31. \end{aligned}$$

Our range of H values are consistent with values used in (Babcock, 2007; Tiffany, 2006) and in our simulations we use the midpoint, $H = 3.09$.

To add to the realism of this model, we also consider the fact that ethanol production is highly subsidized. Thus, we add a constant term (in dollars/gallon) to the gasoline price, which accounts for the volumetric tax credit given to ethanol producers. In other words, for each gallon of ethanol produced at a given facility in the US, the facility receives a subsidy in the form of a tax credit. This subsidy is around $s = 60$ cents per gallon of ethanol produced (Koplow, 2006). We must also account for the cost of running the plant. Some plants in the US use coal and even methane from cow manure to run their plants, and one US ethanol plant uses excess steam from a nearby power plant. However, most plants rely on natural gas, and the associated cost is approximately $p = 52$ cents/gallon of ethanol produced (Babcock, 2007).

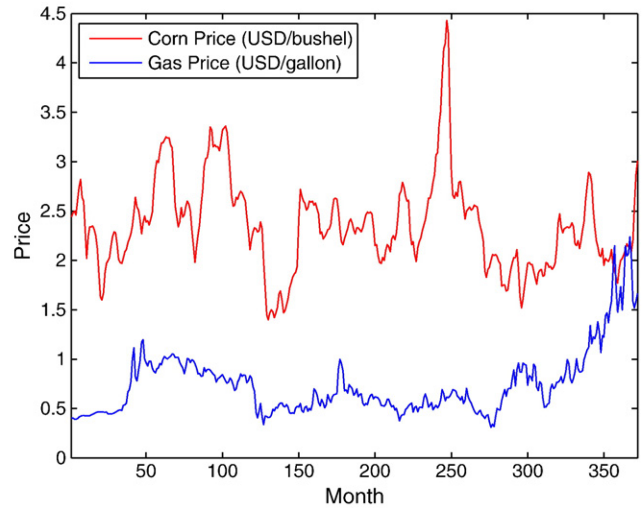

Fig. 2. Corn and gasoline prices, January 1976–December 2006.

The model, which represents the annual value of the plant, is therefore given as

$$\nu \sum_{t=1}^{12} \max(H(G_t + s - p) - C_t, 0), \tag{4}$$

where ν is the number of bushels of corn consumed by the plant per month. Monthly units are used since we are using average monthly corn and gasoline price data. The source for this data is the CRB Commodity Yearbook (2007).

4. Retrospective analysis

We first look at historical corn and gas prices as shown in Fig. 2 (Commodity Research Bureau, 2007). Although this figure depicts 31 years of historical data, we will concentrate on data between January 1997 and December 2006 since it more indicative of the current markets, especially in the case of gasoline.

Fig. 3 uses Eq. (4) to show when the plant was profitable and when it operated at a loss (top panel), on a monthly basis, between January 1997 and December 2006. A value of one on the y-axis represents a profit during a particular month, and a value of zero represents a loss.

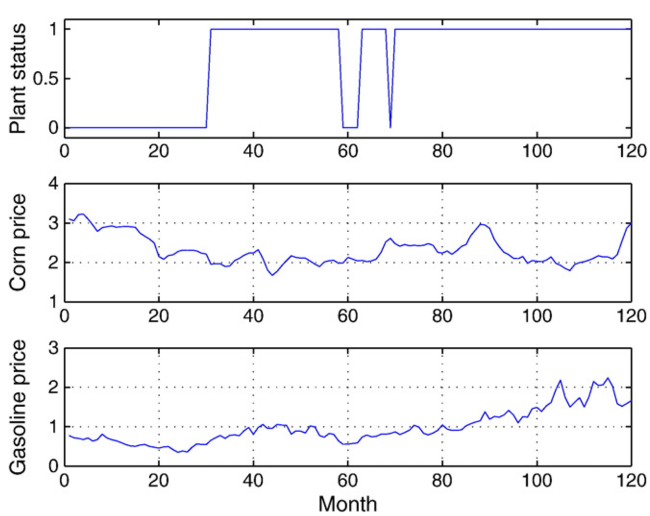

Fig. 3. The bottom panel gives corn price from January 1997 to December 2006 in USD/bushel; middle panel gives gas price, over same time period, in USD/gallon. Top panel indicates whether plant should run (in which case a 1 is plotted) or be turned off (in which case a zero is plotted).

Table 1
Average annual spark spread.

Year	Plant value ($ million)
1997	$0
1998	$0
1999	$2.94
2000	$13.48
2001	$7.71
2002	$2.95
2003	$7.64
2004	$17.67
2005	$41.57
2006	$47.75
Average Annual Plant Value	$14.17
Number of Months plant is running	85 out of 120

Corresponding monthly corn and gasoline spot price data, respectively, are shown in the bottom panels. All prices are in 2006 dollars.

From this figure it is evident that during the last three years of data, under the assumptions given in Section 2, an ethanol plant should be operating at full capacity every month without incurring any losses. This is due to a major increase in gasoline prices during those years, without a corresponding large increase in corn prices. Assuming our future correlation assumption holds, this situation will be highly unlikely as corn and gasoline markets will tend to go up and down together. In fact, since 2006 corn prices have increased substantially so these last three years of data may not be indicative of the current value of an ethanol plant. We look at how to deal with this in the next section.

Table 1 shows the average annual value of the plant from 1997 until 2006. The overall average annual value and the total number of months that the plant operated without incurring losses, assuming Eq. (4) holds, are also given.

5. Corn and gasoline price dynamics

Since past data is not indicative of the current situation, and may not be indicative of the future situation, we can only consider it as one possible scenario. In order to examine multiple scenarios, we must better understand the dynamics of corn and gasoline prices. To do this, we examine the data for seasonality, mean reversion and serial autocorrelation.

We first wish to remove any seasonality which exists in the commodity prices. In order to do this, we look for a sine wave of period 12 that best fits the data (see Haberman, 2004 for details on

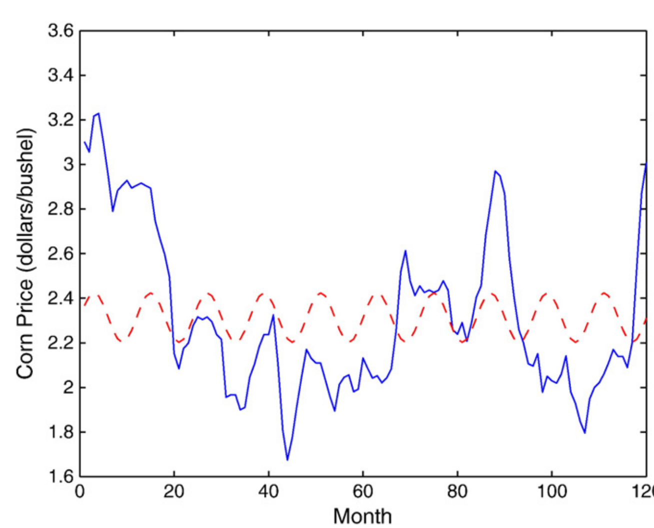

Fig. 4. Average monthly corn price from January 1997 to December 2006, USD/bushel, with best fit sinusoid.

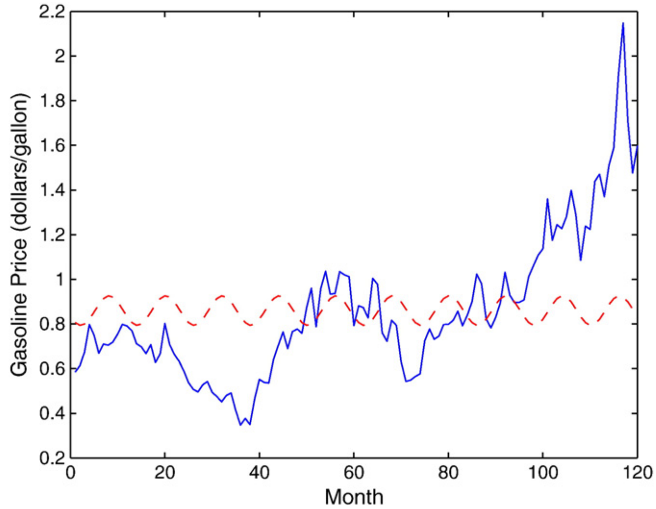

Fig. 5. Average monthly gas price from January 1997 to December 2006, USD/gallon, with best fit sinusoid.

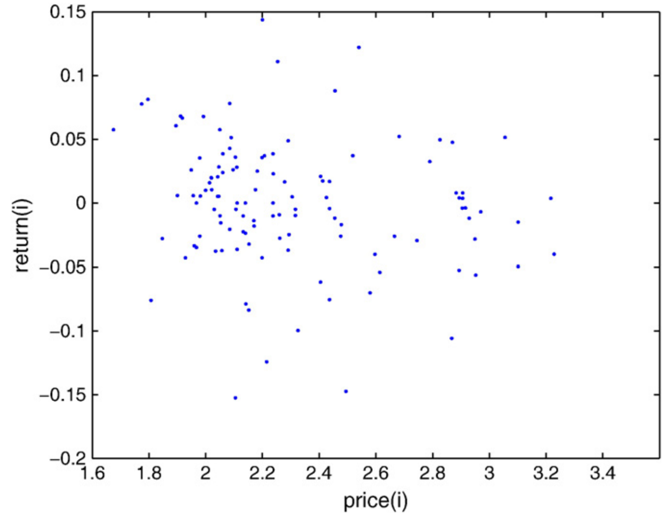

Fig. 6. Log Return(month *i*) *vs.* Price(month *i*) for corn (January 1997 to December 2006).

this). This is done for both data sets, and from Figs. 4 and 5 it is easily shown that the best fit sine waves are not representative of the commodity data. To examine this even further, we determine in which month corn prices are highest and lowest, and compute the probabilities of k highs (or lows) in a given month assuming a completely random draw. The results are given in Tables 2 and 3.

Although obtaining four highs in January is extremely unlikely (it is 0.6% likely to occur at random), there is also a 7% chance that one of the 12 months will show up as an outlier. This, combined with the apparent randomness of annual lows and the fact that the amplitude of the sine wave is not reflective of the data range, we can conclude that the effects of seasonality are at best weak in the corn data and are nonexistent in the gas data; in this study we ignore seasonality entirely.

We also consider the possibility of mean reversion in the data. Figs. 6 and 7 show returns versus prices for both corn and gasoline. If there was significant mean reversion we would expect the returns to be higher than average when prices are very low and lower than average when prices are very high. This is difficult to detect from these figures, and determining a reliable mean reversion parameter would be an even more difficult task, so we also neglect mean reversion in this study.

The final check is for serial autocorrelation in the data. To inspect this, we plot log return $i+1$ versus log return i, shown in Fig. 8 . We find the line of best fit through the data is

$$y = 0.3271x + 0.0007157 \tag{5}$$

with $R^2 = 0.1074$. This value is an indication of how well the regression line approximates the real data points. If we consider only the data from the past 10 years, the line of best fit is

$$y = 0.4311x + 0.0001564 \tag{6}$$

with $R^2 = 0.1845$. In both cases, this is a strong signal, suggesting that there is a linear correlation in the corn data. Thus, we compute the correlation coefficient and Fisher z statistic to confirm this. In general, the correlation coefficient between two data sets X and Y is defined as

$$\rho = \frac{\mathrm{Cov}(X, Y)}{\sqrt{\mathrm{Var}(X)\mathrm{Var}(Y)}}. \tag{7}$$

Table 2
Probability of k highs assuming a random draw.

k	P(k highs)
0	0.4189
1	0.3808
2	0.1558
3	0.0378
4	0.006

Table 3
Frequency of high/low prices.

Month	Number of highs	Number of lows
January	4	1
February	0	0
March	0	0
April	2	1
May	2	0
June	0	1
July	0	1
August	0	1
September	1	1
October	0	2
November	0	1
December	1	1

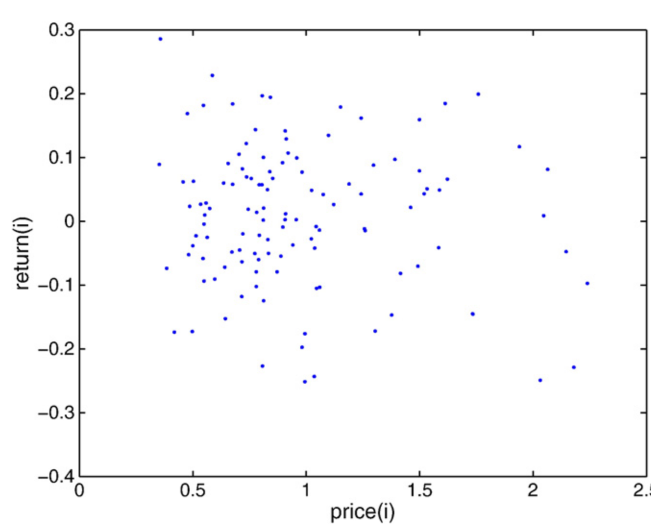

Fig. 7. Log Return(month *i*) *vs.* Price(month *i*) for gas (January 1997 to December 2006).

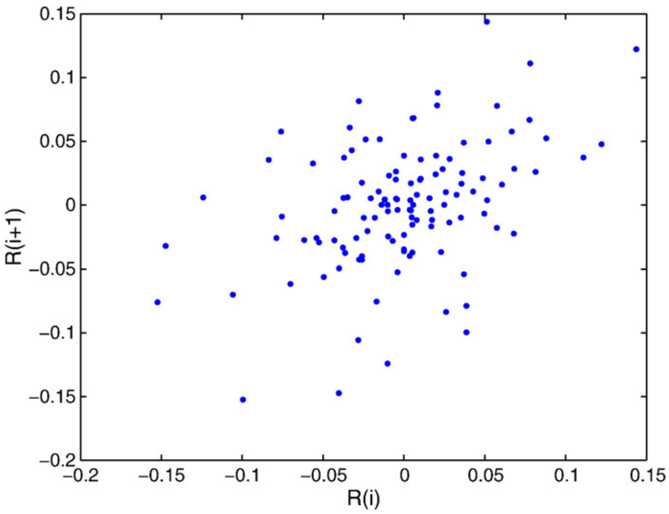

Fig. 8. Log Return$(i+1)$ vs. Log Return(i) for corn (January 1997 to December 2006).

Since X and Y both represent log returns of the same data set, $\mathrm{Var}(X) = \mathrm{Var}(Y)$ and we compute the correlation coefficient to be $\rho_c = 0.4295$. The Fisher z statistic transforms the correlation coefficient into a value which, if the data is actually uncorrelated, is normally distributed with mean zero and standard deviation one, and is given is

$$z_F = \frac{\sqrt{N-3}}{2} ln\left(\frac{1+r}{1-r}\right). \tag{8}$$

This statistic is found to be $z_F = 4.9679$, which means the probability that the log return data is uncorrelated is extremely small and confirms that the corn data has significant autocorrelation.

We redo this procedure using the gasoline spot price data as well. Fig. 9 shows return $i+1$ versus return i, and the line of best fit passing through this data is

$$y = 0.001115x + 0.0005547 \tag{9}$$

with $R^2 = 1.676 \times 10^{-6}$. If we consider only the data from the past 10 years, the line of best fit is

$$y = 0.03148x + 0.006919 \tag{10}$$

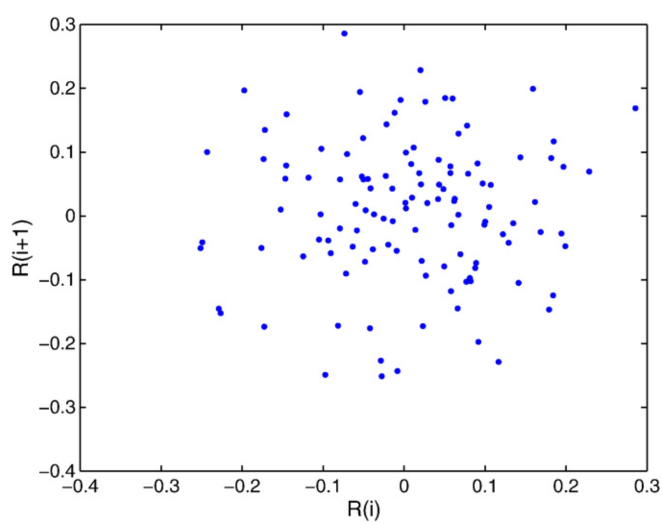

Fig. 9. Log Return$(i+1)$ vs. Log Return(i) for gas (January 1997 to December 2006).

with $R^2 = 0.0009948$. In both cases the regression line is a poor approximation to the actual spot data, and so it appears that the data is random. However, to confirm this we compute the correlation coefficient and Fisher z statistic. We obtain $\rho_g = 0.0315$ and $z_F = 0.3408$, confirming that the data does not have significant autocorrelation.

6. Bootstrap analysis

Now that we better understand the dynamics of corn and gasoline prices, we can generate new sequences of price data, using the "bootstrap" technique. We do this using the log returns from the past ten years. The procedure differs for each commodity due to the autocorrelation in the corn data. For gasoline data, we sample (with replacement) from the log returns to generate a new sequence of log returns. Using the average gasoline price from January 1997 as our initial price value, we can obtain a new price sequence via

$$G^*_{t+1} = G^*_t e^{R_t}, \tag{11}$$

where R_t is determined by sampling from the original sequence of log returns, $R^0_t = log(G_{t+1}/G_t)$.

For corn data we generate an innovation sequence, I_t, using

$$I_{t+1} = \frac{r^0_{t+1} - \alpha r^0_t}{1-\alpha} \tag{12}$$

where $r^0_t = log(C_{t+1}/C_t)$ and $\alpha = \rho_c = 0.4295$. From this sequence we can generate a new sequence, E_t, by sampling with replacement. We can obtain a new price sequence using

$$r_{t+1} = \alpha r_t + (1-\alpha)E_t \tag{13}$$

and

$$C^*_{t+1} = C^*_t e^{r_t}, \tag{14}$$

where $r_1 = E_1$ and C^*_1 is the average corn price from January 1997.

The results are somewhat surprising. The average value of the plant is found to be approximately \$42 million, and the plant is expected to run profitably 64 months out of 120. This means that although the option value is zero for several realizations, other realizations give such large option values that, on average, the plant is extremely profitable. However, a closer look at the simulation values shows that, in several cases, corn and gas prices are unrealistically high or low, and hence the average option value is not a good indication of actual plant value. Thus, we place restrictions on the simulated price values: if corn or gasoline prices exceed a certain value, or fall below a certain value, the simulation is not used. Note that this pruning of unrealistic paths is in some ways similar to the assumption of mean reversion in the return data – mean reversion that was not observable in the data. Mean reversion is notoriously difficult to measure in data sets which, like the ones used here, never move very far from their mean value. Depending on the size of the restriction a large percentage of the simulations may be omitted. For instance, when we use the restrictions $1 < C_t < 15$ and $0.2 < G_t < 10$, about 30% of the simulations are excluded. Using these ranges the average option value is approximately \$25 million and the plant is expected to run at a profit 61 months out of 120. Figs. 10 and 11 are histograms showing the range and frequency of option values for all included simulations as well as the frequency of losses over the 120 month period.

The above technique does not include the possibility of a future correlation between commodity prices. To do this, suppose a bootstrapped sequence of corn and gasoline returns are denoted by r_t and

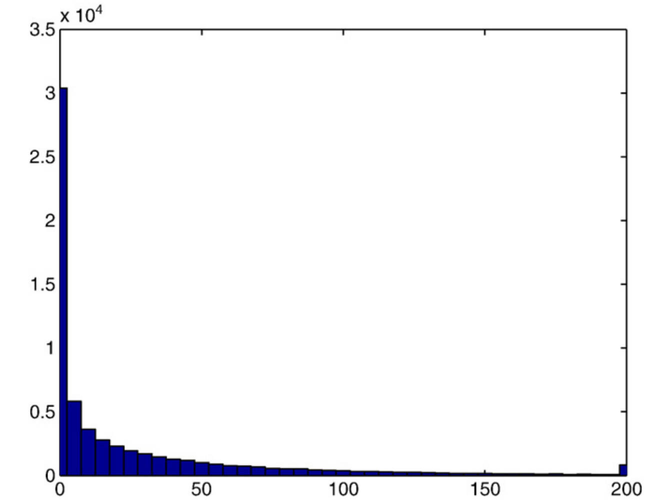

Fig. 10. Frequency of average monthly option values over the 10 year period; 10,000 simulations are used.

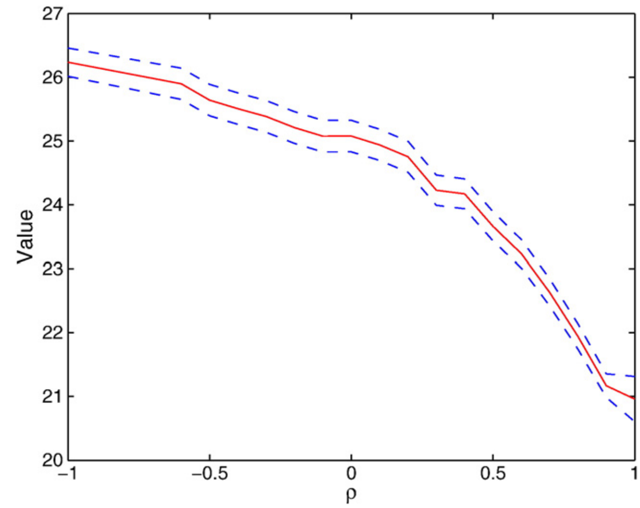

Fig. 12. The solid line represents the average annual plant value with increasing correlation. The dashed lines represent the one standard error range in these calculations.

R_t respectively. We induce correlation between these sequences by creating a new gasoline return sequence R_t^1 as follows:

$$R_t^1 = \left[\rho r_t + (1-\rho)R_t^0\right] \times \frac{\bar{R}_t}{\sqrt{\rho^2\,\bar{r}_t^{\,2} + (1-\rho)^2\,\bar{R}_t^{\,2}}}. \tag{15}$$

Here $\bar{R}_t$ and $\bar{r}_t$ are the average values of the bootstrapped log return sequences, and thus change with each simulation. This means that future gasoline returns do not only depend on past gasoline returns, but they also depend on the corn return in the same period. The amount by which gasoline returns depend on corn returns is the correlation ρ. We can use Eqs. (11) and (14) to obtain new price sequences where the gasoline price at time t depends on both the corn and gasoline log returns from the past. Since we have no way of accurately determining the future correlation of corn and gas prices, we use a variety of ρ values to examine what happens to the value of the plant in these cases, as well as what happens to the number of months that the plant is down. The results are given in Fig. 12. It is evident from this graph that even if corn and gasoline prices become moderately correlated, the value of the plant is substantially lower than when correlation is neglected. For instance, a 50% correlation would mean that profits would be 10% less than if the correlation was zero. Moreover, as correlation increases, the value monotonically decreases, and even more financial losses are expected as the correlation approaches one. If the prices are perfectly correlated, profits would be 30% less than the expected profit given a zero correlation assumption.

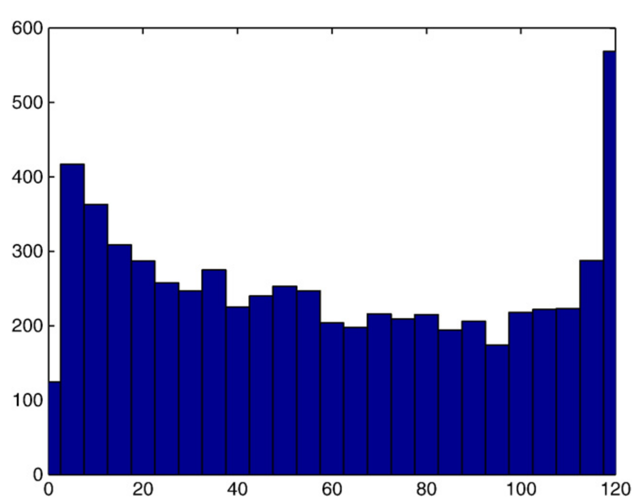

Fig. 11. Frequency of plant losses over the 10 year period; 10,000 simulations are used and losses are computed monthly.

We also look at the number of months the plant is shut down (Fig. 13) and notice that it is generally increasing with increasing correlation. This means that, although the plant is running more, when it is running it is making less profit. As correlation increases, corn and gas prices will tend upward and downward together more often. Situations where corn prices decrease substantially and gas prices increase substantially at the same time are now very unlikely. Thus, the opportunity to take advantage of this large positive spread has been eliminated, and ethanol producers see smaller positive spreads more often.

7. Discussion and conclusions

This paper develops a robust framework for exploring the optimal operation of an ethanol plant. Since historical data may not be an

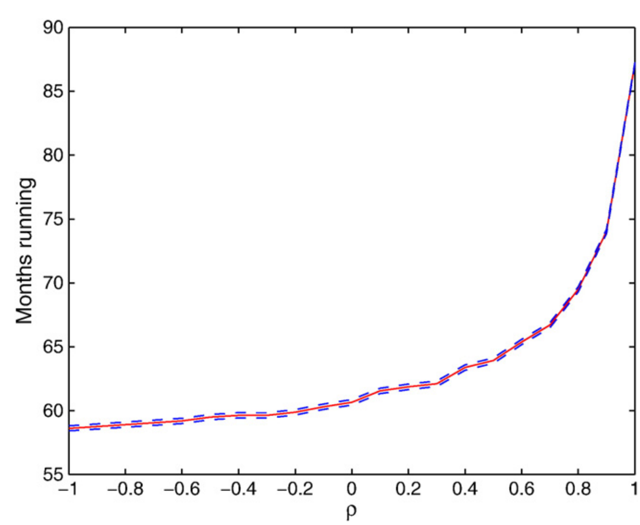

Fig. 13. The solid line represents the number of months the plant is running with increasing correlation. The dashed lines represent the one standard error range in these calculations.

accurate representation of future prices we can consider the time series for corn and gasoline as a single realization, with considerably low corn prices throughout making up for low gas prices in the first several months of the time window. This may explain why the number of months which the plant is not operating (or is operating while incurring losses in the case of historical data) is much lower in this case as compared to values obtained using the bootstrapping techniques.

It is also determined that placing restrictions on the range of acceptable corn and gasoline prices, especially in the bootstrapping technique, causes the average option value to become more reasonable. Mean reverting corn and gasoline prices would imply a similar effect to this truncation of extreme prices. The bootstrapping data relies heavily on the historical data, and since it is uncertain how well past prices will represent future prices, it is also uncertain how reliable the bootstrapping approach is. Thus, we modify this technique to incorporate our prediction of future markets being significantly correlated. It is shown that, if the correlation assumption is correct, the value of an ethanol plant monotonically decreases, the largest decrease occurring when the price moves are highly correlated. Similarly, the number of months the plant is down increases substantially with increasing correlation, indicating that the optimal production schedule heavily depends on the future correlation.

These results can potentially impact the decisions of current plant developers a great deal. In light of these findings, we note that that the subsidy value and the running cost (per gallon of ethanol produced) is assumed constant and fixed. Thus, to ensure that plant owners do not see a decrease in value of their existing and future facilities, per gallon subsidies will have to increase. Moreover, plant owners may be forced to find more cost efficient ways to run their plants. The latter is already being done at a small number of plants in the U.S, as mentioned in Section 2.

Finally, the investigation could be continued by moving away from a data driven valuation to numerically solving low dimensional PDEs with a set of desired assumptions. If the results of both investigations are in agreement then we can state our conclusions more confidently. This will be the focus of future work.

References

Babcock, Bruce A. The Long-Run Impact of Corn–Based Ethanol on the Grain, Oilseed, and Livestock Sectors: A Preliminary Assessment. Presented at the Iowa Pork Congress 2007, Des Moines, Iowa.

The CRB Commodity Yearbook. John Wiley & Sons, Inc.

Deng, S., Johnson, B., Sogomonian, A., 1994. Spark Spread Options and the Valuation of Electricity Generation Assets. Proceedings of the 32nd Hawaii International Conference on System Sciences.

Haberman, Richard, 2004. Applied Partial Differential Equations with Fourier Series and Boundary Value ProblemsFourth Edition. Pearson, New Jersey.

Kim, Seungdo, Dale, Bruce E., 2005. Environmental aspects of ethanol derived from no-tilled corn grain: nonrenewable energy consumption and greenhouse gas emissions. Biomass and Energy 28, 475–489.

Koplow, Doug, 2006. Biofuels — At What Cost? Government Support for Ethanol and Biodiesel in the United States. International Institute for Sustainable Development, Manitoba.

Margrabe, William, 1978. The value of an option to exchange one asset for another. The Journal of Finance 1978 (33), 177–186.

Patzek, Tad W., Anti, S.-M., Campos, R., Ha, K.W., Lee, J., Li, B., Padnick, J., Yee, S.-A., 2005. Ethanol from corn: clean renewable fuel for the future, or drain on our resources and pockets? Environment, Development and Sustainability 7, 319–336.

Pimentel, David, 2003. Ethanol fuels: energy balance, economics, and environmental impacts are negative. Natural Resources Research 12, 127–134.

Pimentel, David, Patzek, Tad W., 2005. Ethanol production using corn, switchgrass, and wood; biodiesel production using soybean and sunflower. Natural Resources Research 14 (1), 65–76.

Shapouri, Hosein, Duffield, James A., Wang, Michael, 2002. The energy balance of corn ethanol: an update, United States Department of Agriculture. Agricultural Economic Report Number 813.

Tiffany, Douglas G., 2006. Fossil Fuels and Ethanol Plant Economics. Presented at Managing High Energy Costs. Redwood Falls, MN.

Wilmott, Paul, 2000. Quantitative Finance, 2. John Wiley & Sons, Ltd, New York.