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Optimal Control of Two-Dam Hydro Facility

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Abstract

This paper investigates optimal control strategies for the operations of a hydroelectric facility comprising two dams linked in series with a common reservoir receiving the outflow from the first dam and supplying the inflow to the second dam. We obtain some interesting insights about the behavior of these plants and compare the similarities and differences of the optimal pump-wait-release cycle between two-dam control and an otherwise similar one-dam control. In particular, we show that the electricity price plays a more important role in the control of two-dam systems than for a single dam.

Keywords: hydroelectric facility, inflow rate, turbine efficiency, head, optimization, optimal control.

1. Introduction

Using hydroelectric dams more efficiently is of major and increasing economic importance in the transition to a low carbon energy economy. In addition, the recent deregulation of many electricity markets has made the optimal control for hydro facilities in the face of variable electricity prices very important. Hydro dams often occur in series along rivers and the resulting connections between dams further complicate these control problem. Investigating the impact of variable prices on series connected dams is the focus of this paper, which extends our earlier work on the optimal control of a single hydroelectric dam. In [3] we discussed how the inflow rate, price and other factors affect the control strategy for a one-dam hydro facility and generalized this system for random water inflows in [4]. For small inflow rates, the control strategy is to maximize the turbine efficiency and variable prices affect the control strategy a great deal. For large inflow rates, the control strategy is to maximize the power function and the variable price does not affect the control strategy. Dynamic programming allows us to determine the optimal control giving the best balance of all factors.

This paper investigates the optimal control of the hydroelectric facility described in Section 2, which comprises two dams linked in series with a common reservoir receiving the outflow from the first dam and supplying the inflow to the second dam. Section 3 casts the problem as a dynamic program analyzed for two different objective functions. In section 4, we determine the optimal control when generating the largest

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amount of energy or, equivalently, for the case in which the electricity price is constant. Section 5 generalizes this analysis to a deterministic, but variable, electricity price in which total sales are optimized. Sections 4 and 5 display and discuss solutions for the optimal control and display solutions taking particular care to discuss the similarities and differences of the resulting optimal pump-wait-release cycle between two-dam control and the control of a similar single dam. Section 6 compares the value of the two dam system with a similar single dam system. We are able to obtain some interesting insights about the behavior of these plants. In particular, we show that the electricity price plays a more important role in two-dam control than in one-dam control, since for a given inflow rate, water is more scarce for two-dam than one-dam case. The paper is summarized in section 7.

2. Two-Dam Model

Consider the problem of controlling a network of three reservoirs and two dams, where each reservoir dam pair is equipped with a pump. As shown in Figure 1, the water discharged from the upper dam flows into the lower reservoir. Now we set up the mathematical model for controlling the flow over these dams in such a way as to maximize the objectives: namely, the total energy generated and the total monetary value of this energy assuming it is sold for a time varying but deterministic price $p(t)$.

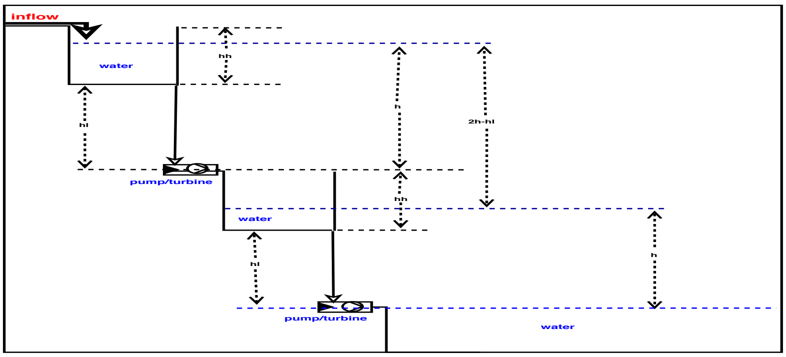

Fig. 1. Two-dam pumped storage plant: head (h), distance between reservoir and turbine (hl), reservoir height (hh), head that upper pump overcomes (2h-hl).

The analysis proceeds as follows: let $V(t, T, p, h^u, h^l)$ be the value of the hydro network. It is determined by,

$$\begin{cases} V_t = \max\limits_{c^u, c^l} \left[\int_t^T e^{-r(s-t)} p E^{ul}(c^u, c^l, h^u, h^l) ds + e^{-r(T-t)} R(h^u_T, h^l_T) \right], \\ dh^u = g^u(s, h^u, f, c^u) ds, \\ dh^l = g^l(s, h^l, c^u, c^l, \tau) ds, \end{cases} \tag{1}$$

where t is the current time and T is the end of the time horizon,
τ is the delay time that water from upper reservoir arrives at the lower reservoir,
p is the electricity price,
c^u and c^l are the controls for the upper and lower reservoir (pump/release),
f is the inflow rate into the upper reservoir,
h^u and h^l are the water head at the upper and lower reservoirs,
r is the discount factor for the time value of money,
$R(h^u_T, h^l_T)$ is the residual value of the water remaining in the reservoir at T.

To keep our analysis tractable, we assume that both reservoirs have the same geometry as in [3], and water discharged from the upper dam immediately appears in the lower reservoir (in future work we hope to relax this assumption), i.e. $\tau = 0$. We can further suppose $dh^u = (f - c^u)/S\,dt$, $dh^l = (c^u - c^l)/S\,dt$, where

S^u and S^l are the surface areas for the upper and lower reservoirs, and rewrite the objective function (1) as

$$\begin{cases} V_t = \max\limits_{c^u, c^l}\left[\int_t^T e^{-r(s-t)}p(s)[E^u(c^u, h^u) + E^l(c^l, h^l)]ds + e^{-r(T-t)}R(h^u(T), h^l(T))\right], \\ dh^u = (f - c^u)/S^u ds, \\ dh^l = (c^u - c^l)/S^l ds. \end{cases} \tag{2}$$

3. Computation Algorithm

From the two-dam objective function (2), we suppose the discount factor r and the residual value $R(h^u_T, h^l_T)$ are zero and no time delay, so in this case the hourly discretized function is

$$\begin{cases} V_t = \max\limits_{c^u_s, c^l_s}\left\{\sum_{s=t}^{T} p_s[E^u(c^u_s, h^u_s) + E^l(c^l_s, h^l_s)]\right\}, \\ h^u_{t+1} = h^u_t + (f_t - c^u_t)/S^u, \\ h^l_{t+1} = h^l_t + (c^u_t - c^l_t)/S^l. \end{cases} \tag{3}$$

If this objective is to be achieved, we must assume that the optimal choices (this is Bellman's principle of dynamic programming [1]) will be made not only at $s = t$ but also at $s = t + 1, t + 2, \cdots, T$. So

$$V_{t+1} = \max_{c^u_s, c^l_s}\left\{\sum_{s=t+1}^{T} p_s[E^u(c^u_s, h^u_s) + E^l(c^l_s, h^l_s)]\right\},$$
$$V_t = \max_{c^u_t, c^l_t}\left\{p_t[E^u(c^u_t, h^u_t) + E^l(c^l_t, h^l_t)] + V_{t+1}\right\}.$$

$V(t, T, p, h^u, h^l)$ is the optimal value at every time step, and it is relatively straight forward to find the control strategy,

$$[c^{u^*}, c^{l^*}] = \arg\max_{c^u_t, c^l_t}\left\{p_t[E^u(c^u_t, h^u_t) + E^l(c^l_t, h^l_t)] + V_{t+1}\right\}.$$

Refer to [3] for a detailed description of the algorithm.

In addition to the same geometric reservoir properties assumed, we also assume that dam 1 and dam 2 are geometrically identical and contain machinery identical to that presented in [3]. The head of the middle reservoir changes as the water is pumped to the upper reservoir, whereas the downstream level does not change when water is pumped into the middle reservoir (see Figure 1). Based on the power function presented in [2] and [3], the two-dam power functions $E^u(c^u, h^u)$ and $E^l(c^l, h^l)$, giving the total power generated measured in megawatts, are

$$E^u(c^u, h^u) = \begin{cases} 10^{-6}\eta\rho g c(h^u - h^u_f), & 0 \le c^u \le \pi\sqrt{2gh^u}, \quad \text{release or wait}, \\ -\alpha_p, & c^u = \dfrac{-\alpha_p\gamma}{10^{-6}\rho g(2h^u + h^u_f - h_{min})}, \quad \text{pump}, \end{cases} \tag{4}$$

and

$$E^l(c^l, h^l) = \begin{cases} 10^{-6}\eta\rho g c(h^l - h^l_f), & 0 \le c^l \le \pi\sqrt{2gh^l}, \quad \text{release or wait}, \\ -\alpha_p, & c^l = \dfrac{-\alpha_p\gamma}{10^{-6}\rho g(h^l + h^l_f)}, \quad \text{pump}. \end{cases} \tag{5}$$

Where $h_{min} \le h^u \le h_{max}$ and $h_{min} \le h^l \le h_{max}$. Let $\eta_{\max} = 0.85$, $\psi = 60$, $\alpha_p = 15\pi$, $\gamma = 75\%$, $f_f = 0.01$, and $L = 120$ as in [3] and further suppose $h_{\min} = 120$ and $h_{\max} = 180$ for both reservoirs. We discuss the discretized model (3) that the control constraints depend on the head h and use the same hourly price function [2, 3] to explore the optimal control strategies when the inflow rate to the upper reservoir is constant between 0 and 15π.

We solve this model for the deterministic case, give the control path, and discuss the relations for control with head, price, inflow rate, and time for 2 days, i.e. $T = t_0 + 48$.

4. Optimal Energy Operation

First we discuss the simplest possible case, in which the price is constant (to be specific, let $p = 1$). In this special case optimization, profit is equivalent to optimization energy generated from the hydro systems. Obviously the constant price does not affect the control strategy. In this case the optimal pump/release strategy is to release water when it can generate the most energy.

4.1. No Inflow Rate

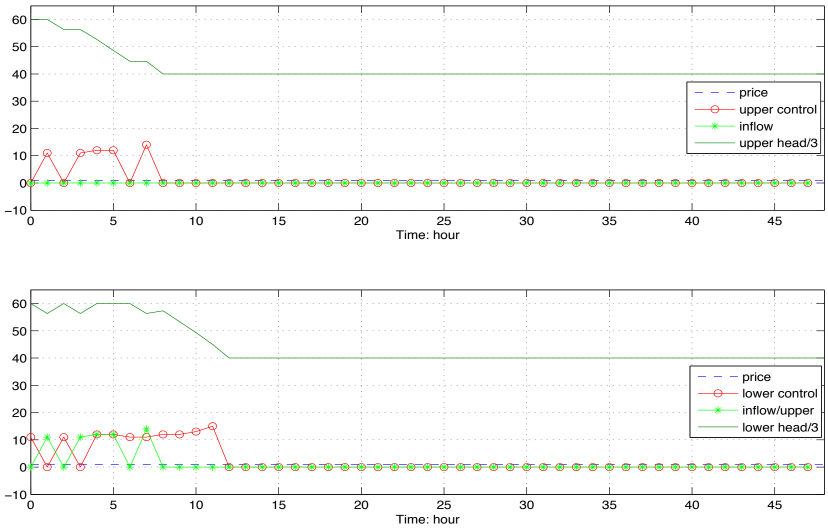

Fig. 2. Maximize energy (both are full, $f = 0$).

We begin with what appears to be a strong special case—no water inflow to the upper dam. Clearly, here the steady state will be to have the minimum allowed water levels in each reservoir after which point nothing can be done. Pumping will never be optimal here because of efficiency losses and the only question is how to extract the maximum energy from the finite irreplaceable water resource. [Note: if the discount rate $r = 0$, this problem does not even have a unique solution, as there is no reason to prefer actions now to actions later]. Maximizing this energy implies running the equipment at its peak efficiency.

We compare this with one-dam control. If both reservoirs are full (Figure 2), the control strategy is to maximize turbine efficiency and so to maximize energy. Here there is some difference from one-dam control in [3]. For the upper dam control, it needs regulations for the lower dam to release water. In other words, we need to release water from the lower reservoir first to make room for water from the upper dam. The control for the lower of two dams is not the same as for an otherwise similar single dam. In order to store more potential energy, the lower reservoir is not empty at the second step. When both reservoirs are empty, do not pump, and both controls remain zero at all subsequent times.

4.2. Different Inflow Rate

When a small inflow is allowed, the problem becomes considerably more complicated and hence more interesting. It still remains the case that the water resource is scarce and must be managed so as to maximize efficiency η instead of the power function E.

Suppose inflow rate $f = 3\pi$ (which is much less than $c^{\eta}_{\max}$ so that the reservoirs cannot be always filled up) we compare the controls for one dam and two dams. For the two-dam control (Figure 3), the upper control needs to release water earlier than the corresponding one-dam control [3] in order to let the lower dam use the water at the final time period.

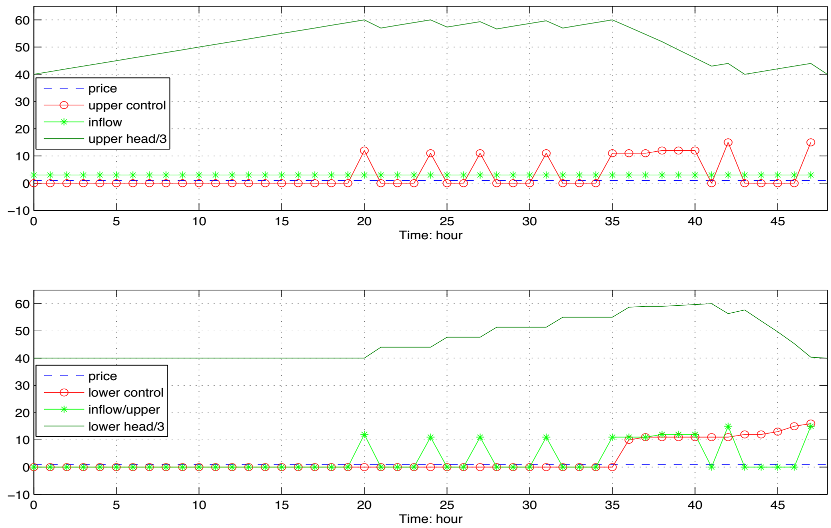

Fig. 3. Two-dam control ($f = 3\pi$).

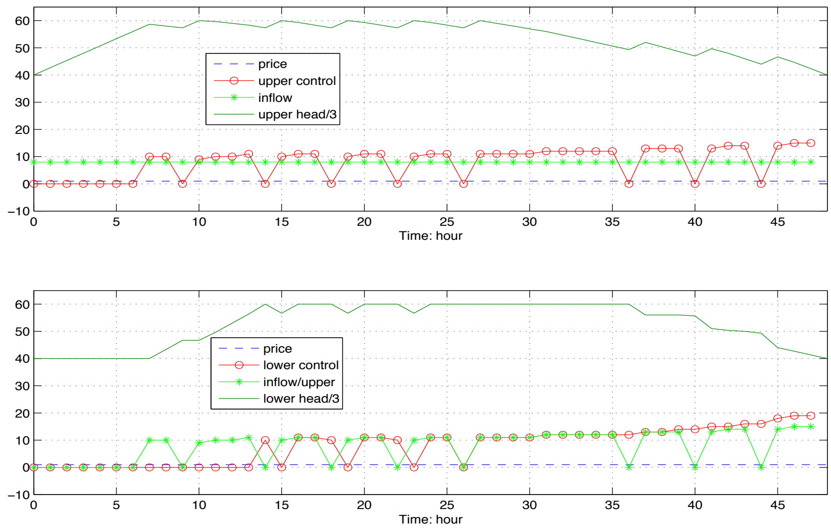

Fig. 4. Two-dam control ($f = 8\pi$).

As the inflow rate increases but less than $c^{\eta}_{\max}$, for example $f = 8\pi$ (Figure 4), the control strategy is same as for one-dam control [3]—store the most potential energy and maximize the turbine efficiency. The pump release patterns at the initial and final transient period adjust a great deal in order to use the water efficiently.

As the inflow rate increases and become large, both controls become equal to the inflow rate (Figure 5) and it is a similar phenomena in the one dam case [3]. This means that the turbine efficiency does not affect the control when the inflow rate can keep both reservoirs full and store the maximum potential energy. It shows that the potential energy becomes more important than the turbine efficiency as the inflow rate increases.

However, notice the difference between Figure 5 and the figure for the same inflow rate in [3]. The final transient is longer in the two-dam case because of the need for the water to be processed first by the upper

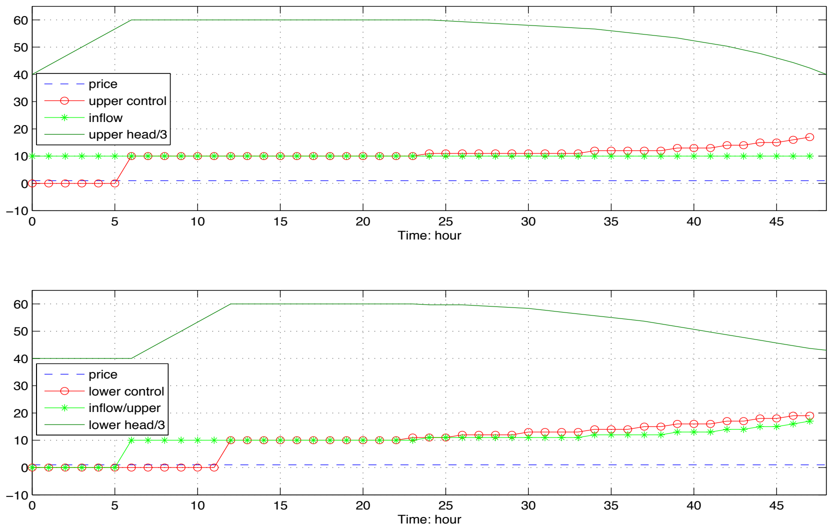

Fig. 5. Two-dam control ($f = 10\pi$).

and then by the lower dam. If the initial heads are in the lowest points, it may be needed to pump water for both controls (depending on the turbine efficiency) in order to increase head to store more potential energy as soon as possible.

As the inflow rate increases enough, some of the water cannot be released from the turbine, both heads are equal to the maximum head, the controls are equal to $c^E_{\max}$ that maximizes the power function E, and the difference for the control only exists at the beginning transient.

4.3. Control and Initial Head

Suppose the inflow rate is $f = 8\pi$, we discuss three cases for the two initial heads: i). Both reservoirs are empty, ii). The upper is full and lower is empty, and iii). Only the lower is full (Figure 6). The controls (release rate and frequency) are adjusted to use the water efficiently according to the initial head, and the strategies are around the value $c^\eta_{\max}$ that maximizes the turbine efficiency before the final time. At first, the controls are different because of the distinct initial heads, and they gradually adjust to similar value and pattern. The control for the upper reservoir is to maximize turbine efficiency (because water is scarce) and release as much water at the second last time period. If the reservoir is empty and inflow rate is small, the lower control acts to pump some water, adjust the release rate, and release as much water at the last time period.

If the inflow rate can make c maximize power function E, the control strategy is to keep the E maximum, and the controls are only different at an initial period transient for various initial heads. This can be proved by:

$$
\begin{aligned}
V &= \max_{c^u,c^l} \int_t^T e^{-r(s-t)} p(s)[E^u(c^u, h^u) + E^l(c^l, h^l)]ds \\
&\leq 2 \int_t^T e^{-r(s-t)} p(s) E_{\max} ds \\
&= 2E_{\max} \int_t^T e^{-r(s-t)} p(s) ds.
\end{aligned}
$$

here $E_{\max}$ is constant, $p(s) = 1$, and $c^u = c^l = c^E_{\max}$.

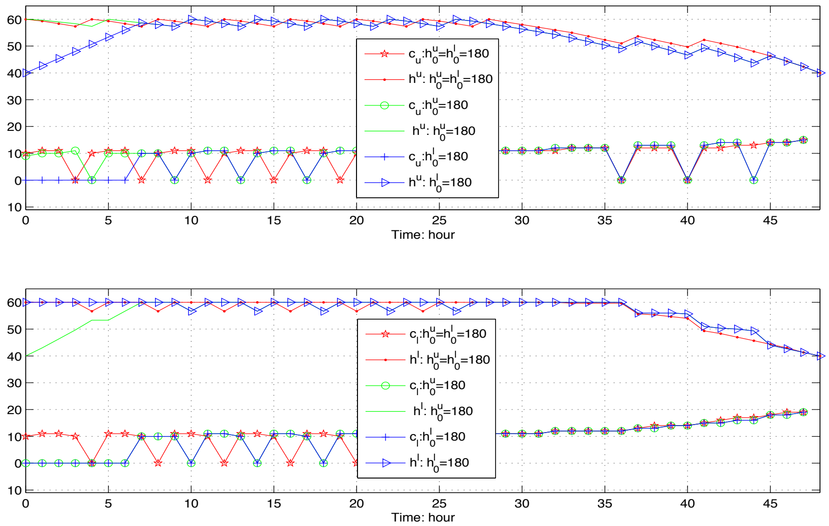

Fig. 6. Different initial head ($f = 8\pi$).

5. Optimal Profit Operation

In what discussed above, we have been concerned with maximizing the total electrical energy generated by two dams in a given time. Instead, we might want to maximize the value of the energy so generated. This value maximizing problem is affected by the electricity price. Of course, with a constant price as in the previous section these two are equivalent. In this section, we investigate optimal control in the case where the price is not constant. Instead, it is a sinusoidal deterministic function. If there is no inflow, a pump and release strategy to maximize cash value is to maximize the turbine efficiency during high price (this is the same as in [2]); only when the release generates as much energy as possible for a given amount of water (that is pumped during low price), can the control maximize the cash value. If the inflow rate is much greater, the optimal control is the same as maximizing energy except that control is different until power generation E becomes maximum. Now we discuss the control with initial head and inflow rate separately for two dams.

5.1. No Inflow

In the first case we assume no water inflow. Because the price is variable, pump and release strategies become possible. In Figure 7, there is pumping for both controls. Compare with the one-dam control presented in [3], two-dam controls pump more water so as to take the advantage of the price difference. However, the upper control releases water a little earlier than the corresponding one-dam control before the maximum price in order to let the lower control also release at high price.

It is interesting that the optimal control may pump for the lower reservoir when the price is low, but it may stop sometimes when the price is lower. At first, this seems illogical, actually this is reasonable, because the upper reservoir also needs to pump water from the lower reservoir, and there should be the exact amount of water to fill up the reservoir. We suppose that pumping water does not overfill the reservoir and the lower control must have previously raised enough water for the upper reservoir to pump.

5.2. Different Inflow Rate

If the inflow rate $f = 3\pi$, there are also pumps for both controls, and the release rate is to maximize turbine efficiency η instead of power function E. With different initial heads or time, the control strategy is to adjust the release and pump based on the price. The time of the upper pumping is shorter than the lower one (Figure 8).

If the inflow rate $f = 6\pi$ (Figure 9), the upper control only pumps at the beginning time; this is because of the losing engineering efficiency. After the lower reservoir is filled, at first both controls are the same, but

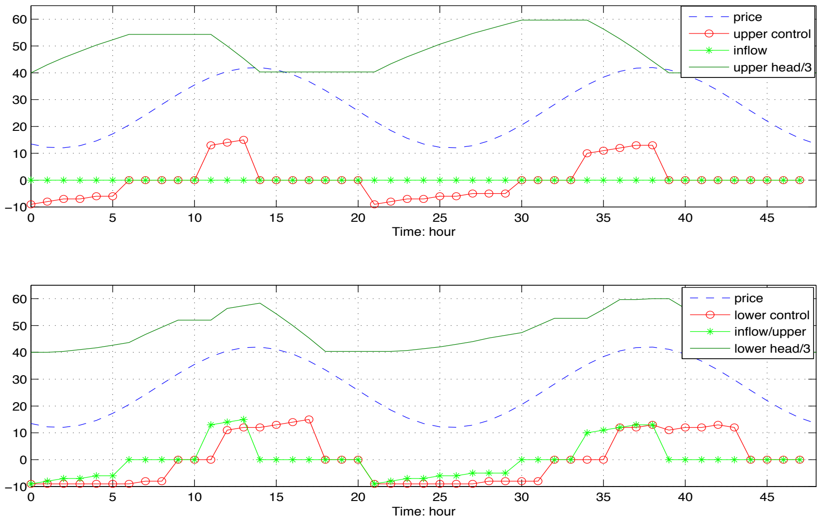

Fig. 7. Maximize value ($f = 0$).

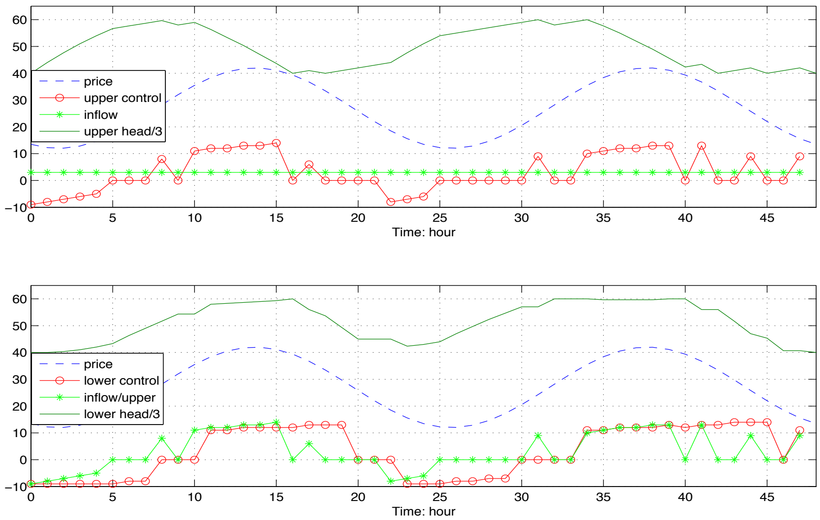

Fig. 8. Maximize value ($f = 3\pi$).

there is some difference before the price is lower. There is much water stored in the lower reservoir, and this water should be released to generate more value at higher price. Pump is needed for the lower control when the price decreases fast in order to increase potential energy. However, there is no-pump after the beginning time for one-dam control in [3]. For the two-dam control, the outflow from the upper reservoir as the inflow rate to the lower reservoir can be adjusted so as to use the pumped water and generate more value. As the inflow rate increases, both controls become similar (Figure 10) but are also affected by the price.

As the inflow rate increases enough ($f = 14\pi$, Figure 11), both reservoirs can be filled up and at the same time the release rates maximize the power function E, and the difference only exists at the beginning transient. Here the inflow rate that makes $c_u = c_l = f$ is greater than that maximizing electricity because of the price impact.

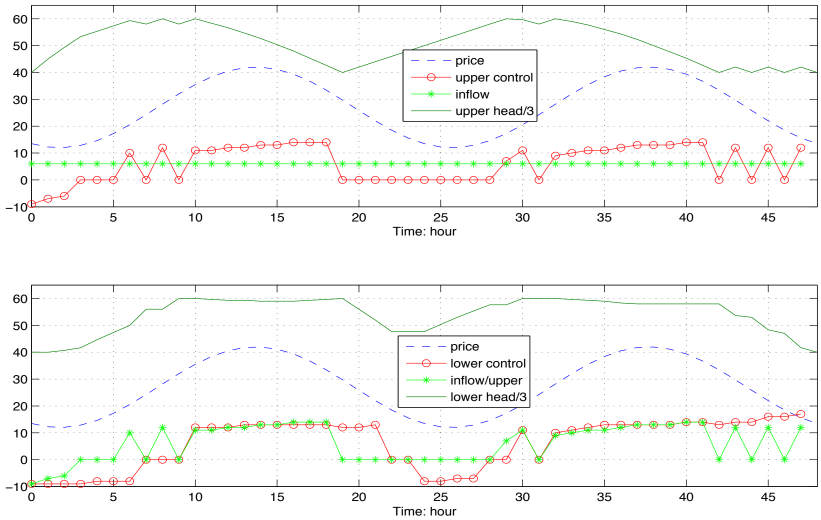

Fig. 9. Maximize value ($f = 6\pi$).

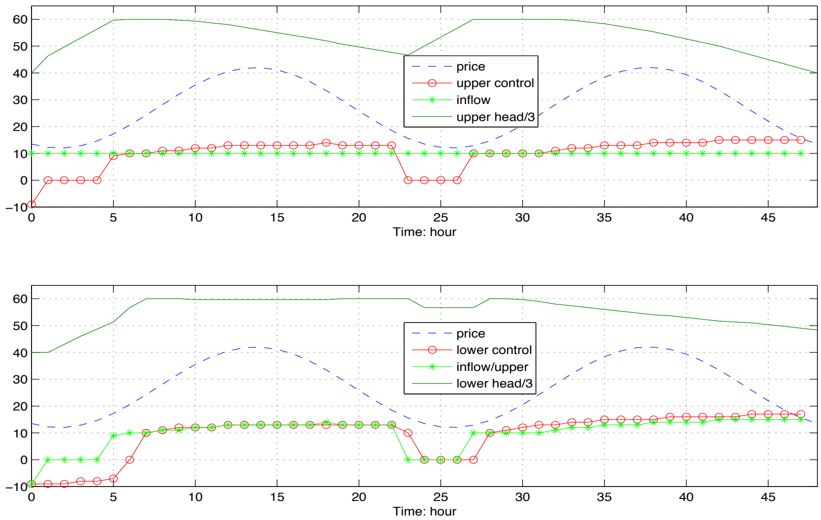

Fig. 10. Maximize value ($f = 10\pi$).

5.3. Control and Initial Head

Suppose the inflow rate is $f = 6\pi$, we plot the controls of the other three cases for the two initial heads in Figure 12: i) Both reservoirs are full, ii) The upper is full and lower is empty, and iii) Only the lower is full. Compare Figure 12 with 9 in which both reservoirs are empty, we see that the controls adjust with the change of the initial head so as to maximize the generated value. Pump little and release more water according to the price-change if the initial head increases. The control adjustment finishes after the first low-high price period, and after that the control strategies are same whatever the initial heads are. Especially, if the inflow rate is large enough, both controls are equal to $c^E_{\max}$ and the same as maximizing energy. For this analysis refer to [3] and Section 4.3 except that the price affects the control at the beginning here.

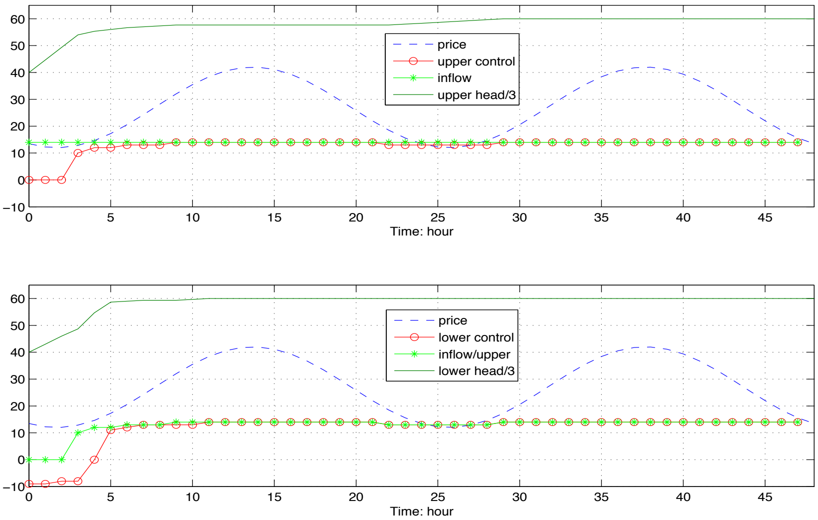

Fig. 11. Maximize value ($f = 14\pi$).

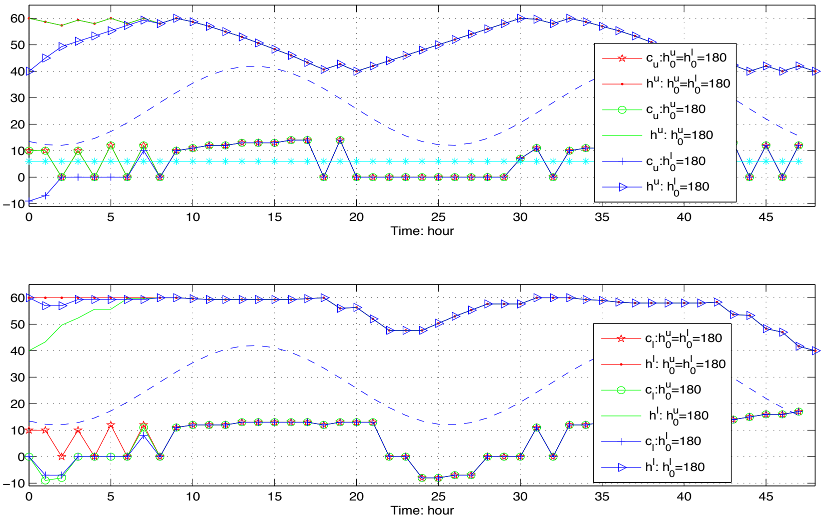

Fig. 12. Different initial head ($f = 6\pi$).

6. Generated Value Comparison

In order to understand the two-dam and one-dam control strategies, this section compares the generated value with respect to head and inflow rate for electricity and cash value respectively.

The ratio of generated value for two-dam over to one-dam increases as the initial head becomes high (see Figure 13), since we have more volume of water to adjust the control based on the power function and price (for cash value). For low initial head, the two-dam generates less electricity than the two independent one-dam. When the initial head is lower, we have less flexibility to adjust the release rate, so the ratio is less than two. However, for the cash value, control for two-dam has much space to pump or release according to the price change, so the ratio is greater than two.

In order to understand the adjustable flexibility, see Figure 14, which shows the generated value vs. the upper and low dam's initial head. For the same initial head, the high upper dam's initial head definitely produces more value, this is because of not only more potential energy bust also more flexible control strategy. For example, the curve for upper dam's initial head with 180 is much higher than that for the same

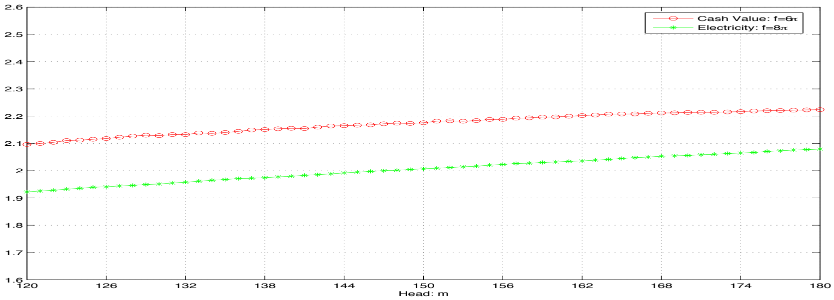

Fig. 13. Ratio of generated value: two dam to one dam vs. initial head.

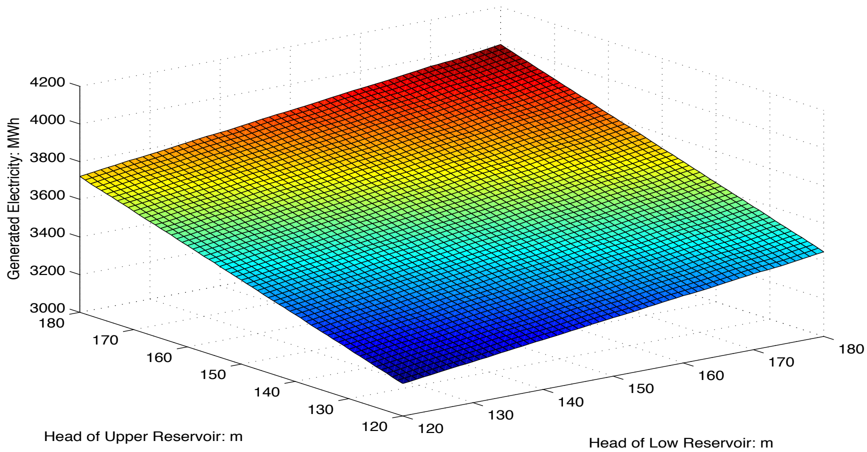

Fig. 14. Generated electricity for 2-dam vs. head ($f = 8\pi$).

lower dam's initial head.

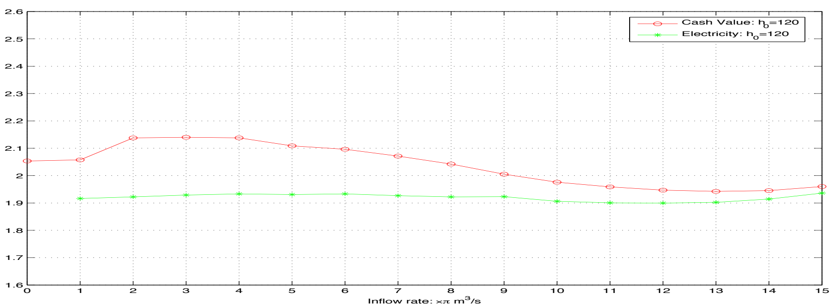

Fig. 15. Ratio of generated value: two dam to one dam vs. inflow rate ($h_0 = 120$).

For the generated value vs. inflow rate, when both reservoirs are empty, the ratio of generated electricity is less than two whatever the inflow rate. Since the price does not affect the control, control is less flexible, and especially for the initial time period, there is much waiting time for the second reservoir to be filled up, so the ratio is less than two. However, for the cash value with small inflow rate, two dam is better than double separated one dam, and as the inflow rate increases, this advantage disappears. This paper uses the 48 hours time period, if the time period is long enough, the ratio will be closed to two (see Figure 15). On the other hand, if both reservoirs are full, two-dam values are greater than two independent one-dam for both cash value and electricity, since there exists more flexibility to control the value generation (see Figure 16). As the inflow rate increases big enough so that the power function is maximized, the ratios are equal to two. The ratio depends on the control flexility difference between one-dam and two-dam, so for the low initial

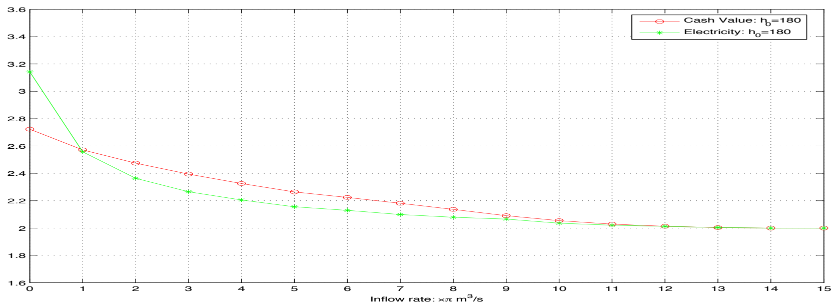

Fig. 16. Ratio of generated value: two dam to one dam vs. inflow rate (h_0 = 180).

head with small inflow rate, the ratio for generated cash value is greater than that for generated electricity; while for the high head with no water inflow, the the ratio for generated electricity is much bigger than for cash value.

7. Summary

This paper analyzed the optimal pump-hold-release strategy for two dams. It showed that this strategy, just as the optimal strategy for the control of a single dam, was affected by reservoir geometry and water inflow rate, the turbine efficiency, the power function, the time horizon, and the price behavior as well as on constraints on water flow rates and water levels. We show that the resulting control strategies are more sensitive to reservoir geometry and the rate of price changes than the otherwise similar one dam control. Generally speaking, the price, the turbine efficiency, and the potential energy each have a different impact on the control for different inflow rates and are more important for the two dam than for the single dam problem. In the scarce water, low inflow rate case, the optimal control uses the water very efficiently, releasing it only at high prices and at a rate designed to optimize turbine efficiency considering potential energy effects.

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