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# Investor Psychology and Security Market Under- and Overreactions

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## Abstract

We propose a theory of securities market under- and overreactions based on two well-known psychological biases: investor overconfidence about the precision of private information; and biased self-attribution, which causes asymmetric shifts in investors' confidence as a function of their investment outcomes. We show that overconfidence implies negative long-lag autocorrelations, excess volatility, and, when managerial actions are correlated with stock mispricing, public-event-based return predictability. Biased self-attribution adds positive short-lag autocorrelations ('momentum'), short-run earnings 'drift,' but negative correlation between future returns and long-term past stock market and accounting performance. The theory also offers several untested implications and implications for corporate financial policy.

In recent years a body of evidence on security returns has presented a sharp challenge to the traditional view that securities are rationally priced to reflect all publicly available information. Some of the more pervasive anomalies can be classified as follows (Appendix A cites the relevant literature):

1. Event-based return predictability (public-event-date average stock returns of the same sign as average subsequent long-run abnormal performance)

2. Short-term momentum (positive short term autocorrelation of stock returns, for individual stocks and the market as a whole)

3. Long-term reversal (negative autocorrelation of short-term returns separated by long lags, or "overreaction")

4. High volatility of asset prices relative to fundamentals.

5. Short-run post-earnings announcement stock price 'drift' in the direction indicated by the earnings surprise, but abnormal stock price performance in the opposite direction of long-term earnings changes.

There remains disagreement over the interpretation of the above evidence of predictability. One possibility is that these anomalies are chance deviations to be expected under market efficiency (Fama (1998)). We believe the evidence does not accord with this viewpoint because some of the return patterns are strong and regular. The size, book-to-market, and momentum effects are present both internationally and in different time-periods. Also, the pattern mentioned in (1) above obtains for the great majority of event studies.

Alternatively, these patterns could represent variations in rational risk-premia. However, based on the high Sharpe ratios (relative to the market) apparently achievable with simple trading strategies (MacKinlay (1995)), any asset pricing model consistent with these patterns would have to have extremely variable marginal utility across states. Campbell and Cochrane (1994) find that a utility function with extreme habit persistence is required to explain the predictable variation in market returns. To be consistent with cross-sectional predictability findings (on size, book-to-market, and momentum, for example), a model would presumably require even more extreme variation in marginal utilities. Also, the model would require that marginal utilities covary strongly with the returns on the size, book-to-market and momentum portfolios. No such correlation is obvious in examining the data. Given this evidence, it seems reasonable to consider explanations for the observed return patterns based on imperfect rationality.

Moreover, there are important corporate financing and payout patterns which seem potentially related to market anomalies. Firms tend to issue equity (rather than debt) after rises in market value, and when the firm or industry book/market ratio is low. There are industry-specific financing and repurchase booms, perhaps designed to exploit industry-level mispricings. Transactions such as takeovers that often rely on securities financing are also prone to industry booms and quiet periods.

Although it is not obvious how the empirical securities market phenomena can be captured plausibly in a model based on perfect investor rationality, no psychological ("behavioral") theory for these phenomena has won general acceptance. Some aspects of the patterns seem contradictory, such as apparent market underreaction in some contexts and overreaction in others. While explanations have been offered for particular anomalies, we have lacked an integrated theory to explain these phenomena, and out-of-sample empirical implications to test proposed explanations.

A general criticism often raised by economists against psychological theories is that, in a given economic setting, the universe of conceivable irrational behavior patterns is essentially unrestricted. Thus, it is sometimes claimed that allowing for irrationality opens a Pandora's box of *ad hoc* stories which will have little out-of-sample predictive power. However, DeBondt and Thaler (1995) argue that a good psychological finance theory will be grounded on psychological evidence about how people actually behave. We concur, and also believe that such a theory should allow for the rational side of investor decisions. To deserve consideration a theory should be parsimonious, explain a *range* of anomalous patterns in different contexts, and generate new empirical implications. The goal of this paper is to develop such a theory of security markets.

Our theory is based on investor *overconfidence*, and variations in confidence arising from *biased self-attribution*. The premise of investor overconfidence is derived from a large body of evidence from cognitive psychological experiments and surveys (summarized in Section I) which shows that individuals overestimate their own abilities in various contexts.

In financial markets, analysts and investors generate information for trading through means, such as interviewing management, verifying rumors, and analyzing financial statements, which can be executed with varying degrees of skill. If an investor overestimates his ability to generate information, or to identify the significance of existing data which others neglect, he will underestimate his forecast errors. If investors are more overconfident about signals or assessments with which they have greater personal involvement, they will tend to be overconfident about the information they themselves have generated but not about public signals. Thus, we define an *overconfident* investor as one who overestimates the precision of his private information signal, but not of information signals publicly received by all.

We find that the overconfident informed overweigh the private signal relative to the prior, causing the stock price to overreact. When noisy public information signals arrive, the inefficient deviation of the price is partially corrected, on average. On subsequent dates, as more public information arrives, the price, on average, moves still closer to the full-information value. Thus, a central theme of this paper is that stock prices overreact to private information signals and underreact to public signals. We show that this overreaction-correction pattern is consistent with long-run negative autocorrelation in stock returns, unconditional excess volatility (unconditional volatility in excess of that which would obtain with fully rational investors), and with further implications for volatility conditional on the type of signal.

The market's tendency to over- or under-react to different types of information allows us to address the remarkable pattern that the average announcement date returns in virtually all event studies are of the same sign as the average post-event abnormal returns. Suppose that the market observes a public action taken by an informed party such as the firm at least partly in response to market mispricing. For example, a rationally managed firm may tend to buy back more of its stock when managers believe their stock is undervalued by the market. In such cases, the corporate event will reflect the manager's belief about the market valuation error, and will therefore predict future abnormal returns. In particular, repurchases, reflecting undervaluation, will predict positive abnormal returns, while equity offerings will predict the opposite. More generally, actions taken by any informed party (such as a manager or analyst) in a fashion responsive to mispricing will predict future returns. Consistent with this implication, many events studied in the empirical literature can reasonably be viewed as being responsive to mispricing, and have the abnormal return pattern discussed above. Subsection II.B.4 offers several additional implications about the occurrence of and price patterns around corporate events and for corporate policy that are either untested or have been confirmed only on a few specific events.

The empirical psychology literature reports not just overconfidence, but that as individuals observe the outcomes of their actions, they update their confidence in their own ability in a biased manner. According to *attribution theory* (Bem (1965)), individuals too strongly attribute events that confirm the validity of their actions to high ability, and events that disconfirm the action to external noise or sabotage. (This relates to the notion of cognitive dissonance, in which individuals internally suppress information that conflicts with past choices.)

If an investor trades based on a private signal, we say that a later public signal *confirms* the trade if it has the same sign (good news arrives after a buy, or bad news after a sell). We assume that when an investor receives confirming public information, his confidence rises, but disconfirming information causes confidence to fall only modestly, if at all. Thus, if an individual begins with unbiased beliefs, new public signals *on average* are viewed as confirming the private signal. This suggests that public information can trigger further overreaction to a preceding private signal. We show that such continuing overreaction causes momentum in security prices, but that such momentum is eventually reversed as further public information gradually draws the price back toward fundamentals. Thus, biased self-attribution implies short-run momentum and long-term reversals.

The dynamic analysis based on biased self-attribution can also lead to a lag-dependent response to corporate events. Cash flow or earnings surprises at first tend to reinforce confidence, causing a same-direction average stock price trend. Later reversal of overreaction can lead to an opposing stock price trend. Thus, the analysis is consistent with both short term post-announcement stock price trends in the same same direction as earnings surprises and later reversals.

In our model, investors are quasi-rational in that they are Bayesian optimizers except for their overassessment of *valid* private information, and their biased updating of this precision. A frequent objection to models that explain price anomalies as market inefficiencies is that fully rational investors should be able to profit by trading against the mispricing. If wealth flows from quasi-rational to smart traders, eventually the smart traders may dominate price-setting. However, for several reasons, we do not find this argument to be compelling, as discussed in the conclusion.

Several other papers have modeled overconfidence in various contexts. Hirshleifer, Subrahmanyam, and Titman (1994) examined how analyst/traders who overestimate the probability that they receive information before others will tend to herd in selecting stocks to study. Kyle and Wang (1997), Odean (1998), and Wang (1998) provide specifications of overconfidence as overestimation of information precision, but do not distinguish between private and public signals in this regard (see also Caballé and Sákovics (1996)). Odean (1998) examines overconfidence about, and consequent overreaction to, a private signal. As a consequence there is excess volatility and negative return autocorrelation. Because our model assumes that investors are overconfident only about private signals, we obtain underreaction as well as overreaction effects. Furthermore, because we consider time-varying confidence, there is continuing overreaction to private signals over time. Thus, in contrast with Odean, we find forces toward positive as well as negative autocorrelation; and we argue that overconfidence can decrease volatility around public news events.[1]

Daniel, Hirshleifer, and Subrahmanyam (1998) show that our specification of overconfidence can help explain several empirical puzzles regarding cross-sectional patterns of security return predictability and investor behavior. These puzzles include the ability of price-based measures (dividend yield, earnings/price, book/market, and firm market value) to predict future stock returns, possible domination of $\beta$ as a predictor of returns by price-based variables, and differences in the relative ability of different price-based measures to predict returns.

A few other recent studies have addressed both overreaction and underreaction in an integrated fashion. Shefrin (1997) discusses how base rate underweighting can shed light on the anomalous behavior of implied volatilities in options markets. In a contemporaneous paper, Barberis, Shleifer, and Vishny (1998) offer an explanation for under- and over-reactions based on a learning model in which actual earnings follow a random walk, but individuals believe that earnings follow either a steady growth trend, or else are mean-reverting. Since their focus is on learning about the time-series process of a performance measure such as earnings, they do not address the sporadic events examined in most event studies. In another recent paper, Hong and Stein (1997) examine a setting where under- and over-reactions arise from the interaction of momentum traders and news watchers. Momentum traders make partial use of the information contained in recent price trends, and ignore fundamental news. Fundamental traders rationally use fundamental news but ignore prices. Our paper differs in focusing on psychological evidence as a basis for assumptions about investor behavior.

The remainder of the paper is structured as follows. Section I describes psychological evidence of overconfidence and self-attribution bias. Section II develops the basic model of overconfidence. Here, we describe the economic setting and define overconfidence. We analyze the equilibrium to derive implications about stock price reactions to public versus private news, short- versus long-term autocorrelations, and volatility. Section III examines time-variation in overconfidence, to derive implications about the signs of short-term versus long-term return autocorrelations. Section IV concludes by summarizing our findings, relating our analysis to the literature on exogenous noise trading, and discussing issues related to the survival of overconfident traders in financial markets.

# I. Overconfidence and Biased Self-Attribution

The model we present in this paper relies on two psychological regularities: *overconfidence* and *attribution bias*. In their summary of the microfoundations of behavioral finance, DeBondt and Thaler (1995) state that "perhaps the most robust finding in the psychology of judgment is that people are overconfident." Evidence of overconfidence has been found in several contexts. Examples include psychologists, physicians and nurses, engineers, attorneys, negotiators, entrepreneurs, managers, investment bankers, and market professionals such as security analysts and economic forecasters.[2] Further, some evidence suggests that experts tend to be more overconfident than relatively inexperienced individuals (Griffin and Tversky (1992)). Psychological evidence also indicates that overconfidence is more severe for diffuse tasks (e.g., making diagnoses of illnesses) which require judgment than for mechanical tasks (e.g., solving arithmetic problems); and tasks for which delayed feedback is received, as opposed to tasks which provide immediate and conclusive outcome feedback, such as weather forecasting or horse-racing handicapping (see Einhorn (1980)). Fundamental valuation of securities (forecasting *long-term* cash flows) requires judgement about open-ended issues, and feedback is noisy and deferred. We therefore focus on the implications of overconfidence for financial markets.[3]

Our theory assumes that investors view themselves as more able to value securities than they actually are, so that they underestimate their forecast error variance. This is consistent with evidence that people overestimate their own abilities, and perceive themselves more favorably than they are viewed by others.[4] Several experimental studies find that individuals underestimate their error variance in making predictions, and overweigh their own forecasts relative to those of others.[5]

The second aspect of our theory is biased self-attribution: the confidence of the investor in our model grows when public information in agreement with his information, but it does not fall commensurately when public information contradicts his private information. The psychological evidence indicates that people tend to credit themselves for past success, and blame external factors for failure (Fischoff (1982), Langer and Roth (1975), Miller and Ross (1975), Taylor and Brown (1988)). As Langer and Roth (1975) put it, 'Heads I win, tails it's chance'; see also the discussion of De Long, Shleifer, Summers, and Waldmann (1991).

# II. The Basic Model: Constant Confidence

This section develops the model with static confidence. Section III considers time-varying confidence. Each member of a continuous mass of agents is overconfident in the sense that if he receives a signal, he overestimates its precision. We refer to those who receive the signal as the informed, $I$; and those who do not as the uninformed, $U$. For tractability, we assume that the informed are risk neutral, whereas the uninformed are risk averse.

Each individual is endowed with a basket containing security shares, and a riskfree numeraire which is a claim to one unit of terminal-period wealth. There are 4 dates. At date 0, individuals begin with their endowments and identical prior beliefs, and trade solely for optimal risk-transfer purposes. At date 1, $I$s receive a common noisy private signal about underlying security value and trade with $U$s.[6] At date 2, a noisy public signal arrives, and further trade occurs. At date 3, conclusive public information arrives, the security pays a liquidating dividend, and consumption occurs. All random variables are independent and normally distributed.

The risky security generates a terminal value of $\theta$, which is assumed to be normally distributed with mean $\bar{\theta}$ and variance $\sigma_\theta^2$. For most of the paper we set $\bar{\theta} = 0$ without loss of generality. The private information signal received by *I*s at date 1 is

$$s_1 = \theta + \epsilon, \tag{1}$$

where $\epsilon \sim N(0, \sigma_\epsilon^2)$ (so the signal precision is $1/\sigma_\epsilon^2$). The *U*s correctly assess the error variance, but *I*s underestimate it to be $\sigma_C^2 < \sigma_\epsilon^2$. The differing beliefs about the noise variance are common knowledge to all.[7] Similarly, the date 2 public signal is

$$s_2 = \theta + \eta, \tag{2}$$

where the noise term $\eta \sim N(0, \sigma_p^2)$ is independent of $\theta$ and $\epsilon$. Its variance $\sigma_p^2$ is correctly estimated by all investors.

Our simplifying assumption that all private information precedes all public information is not needed for the model's implications. It is essential that at least some noisy public information arrives after a private signal. The model's implications stand if, more realistically, additional public information precedes or is contemporaneous with the private signal.

Since prices are set by the risk-neutral informed traders, the formal role of the uninformed in this paper is minimal. The rationale for the assumption of overconfidence is that the investor has a personal attachment to his own signal. This implies some other set of investors who do not receive the same signal. Also, similar results will hold if both groups of investors are risk averse, so that both groups influence price. We have verified this analytically in a simplified version of the model. So long as the uninformed are not risk neutral price setters, the overconfident informed will push price away from fully rational values in the direction described here.

## A. Equilibrium Prices and Trades

Since the informed traders are risk neutral, prices at each date satisfy

$$\begin{aligned} P_1 &= E_C[\theta|\theta + \epsilon] && (3) \\ P_2 &= E_C[\theta|\theta + \epsilon, \theta + \eta], && (4) \end{aligned}$$

where the subscript $C$ denotes the fact that the expectation operator is calculated based on the informed traders' confident beliefs. Trivially, $P_3 = \theta$. By standard properties of normal variables (Anderson (1984), Chapter 2)

$$P_1 = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_C^2}(\theta + \epsilon) \tag{5}$$

$$P_2 = \frac{\sigma_\theta^2(\sigma_C^2 + \sigma_p^2)}{D}\theta + \frac{\sigma_\theta^2 \sigma_p^2}{D}\epsilon + \frac{\sigma_\theta^2 \sigma_C^2}{D}\eta, \tag{6}$$

where $D \equiv \sigma_\theta^2(\sigma_C^2 + \sigma_p^2) + \sigma_C^2 \sigma_p^2$.

## B. Implications for Price Behavior

This section examines the implications of static confidence for over- and under-reactions to information and empirical securities returns patterns. Subsection B.1 examines price reactions to public and private information, subsection B.2 examines the implications for price-change autocorrelations, and subsection B.3 examines implications for event-studies. Subsection B.4 discusses some as-yet-untested empirical implications of the model.

### B.1 Overreaction and Underreaction

Figure 1 illustrates the average price path following a positive (upper curve) or negative (lower curve) date 1 private signal (date $3'$ of the graph has not yet been introduced). At this point we focus on the solid lines. The upper curve, an impulse-response function, shows the expected prices conditional on a private signal of unit magnitude arriving at time 1. The thin horizontal line shows the fully rational price level.

Overconfidence in the private signal $\theta + \epsilon$ causes the date 1 stock price to *overreact* to this new information. At date 2, when noisy public information signals arrive, the inefficient deviation of the price is partially corrected, on average. The same is true on subsequent public information arrival dates. We call the part of the impulse response prior to the peak or trough the *overreaction phase*, and the later section the *correction phase*.

This overreaction and correction implies that the covariance between the date 1 price change and the date 2 price change, $\mathrm{cov}(P_2 - P_1, P_1 - P_0)$, is negative. (Appendix B provides detailed expressions for the covariances described here.) Further, the overreaction to the private signal is partially corrected by the date 2 public signal, and fully corrected upon release of the date 3 public signal, so that $\mathrm{cov}(P_3 - P_1, P_1 - P_0) < 0$. This price change reversal arises from the continuing correction to the date 1 overreaction. Finally, the continuing correction starting at date 2 and ending at date 3 causes price changes at the time of and subsequent to the public signal to be positively correlated, so that $\mathrm{cov}(P_3 - P_2, P_2 - P_1) > 0$. We thus have:

**Proposition 1** *If investors are overconfident, then:*

1. *Price moves resulting from private information arrival are on average partially reversed in the long-run.*

2. *Price moves in reaction to the arrival of public information are positively correlated with later price changes.*

The pattern of correlations described in Proposition 1 is potentially testable by examining whether long-run reversals following days with public news events are smaller than reversals on days without such events. The price behavior around public announcements has implications for corporate event studies (see Subsection B.3).

### B.2 Unconditional Serial Correlations and Volatility

Return autocorrelations in well-known studies of momentum and reversal are calculated without conditioning on the arrival of a public information signal. To calculate a return autocorrelation that does not condition on whether private versus public information has arrived, consider an experiment where the econometrician picks consecutive dates for price changes randomly (dates 1 and 2, versus dates 2 and 3). The date 2 and 3 price changes are positively correlated, but the date 1 and 2 price changes are negatively correlated. Suppose that the econometrician is equally likely to pick either pair of consecutive dates. Then the overall autocorrelation is *negative*:

**Proposition 2** *If investors are overconfident, price changes are unconditionally negatively autocorrelated at both short and long lags.*

Thus, the constant-confidence model accords with long-run reversals (negative long-lag autocorrelations) but not with short-term momentum (positive short-lag autocorrelation). However, the short-lag autocorrelation will be positive in a setting where the extremum in the impulse response function is sufficiently smooth, because the negative autocovariance of price changes surrounding a smooth extremum will be low in absolute terms. Such a setting, based on biased self-attribution and outcome-dependent confidence, is considered in Section III.

Since overconfidence causes wider swings at date 1 away from fundamentals, it causes excess price volatility around private signals ($\mathrm{var}(P_1 - P_0)$) as in Odean (1998). Greater overconfidence also causes relative underweighing of the public signal, which tends to reduce date 2 variance. However, the wide date 1 swings create a greater need for corrective price moves at dates 2 and 3, so that greater overconfidence can either decrease or increase the volatility around public signals ($\mathrm{var}(P_2 - P_1)$). (Explicit expressions for the variances of this section are contained in Appendix B.)

Consider again an econometrician who does not condition on the occurrence of private or public news arrival. He will calculate price change variances placing equal weights on price changes $P_1 - P_0$, $P_2 - P_1$, and $P_3 - P_2$. The unconditional volatility is therefore just the arithmetic mean of $\text{var}(P_3 - P_2)$, $\text{var}(P_2 - P_1)$, and $\text{var}(P_1 - P_0)$. Excess volatility is the difference between the volatility with overconfidence and the volatility when $\sigma_C^2 = \sigma_\epsilon^2$.

Let the subscript $R$ denote the volatility if all individuals were rational. We define the date $t$ proportional excess volatility as

$$V_t^E \equiv \frac{\text{var}(P_t - P_{t-1}) - \text{var}_R(P_t - P_{t-1})}{\text{var}_R(P_t - P_{t-1})}. \tag{7}$$

**Proposition 3** *1. Overconfidence increases volatility around private signals, can increase or decrease volatility around public signals, and increases unconditional volatility.*

*2. The proportional excess volatility is greater around the private signal than around the public signal.*

Thus, consistent with the findings of Odean (1998), when there are only private signals, there is a general tendency for overconfidence to create excess volatility. Excess volatility is not an automatic implication of any model with imperfect rationality. For example, if investors are *underconfident*, $\sigma_C^2 > \sigma_\epsilon^2$, then there will be *insufficient* volatility relative to the rational level. Also, in contrast with Odean, Proposition 3 implies that in samples broken down by types of news event, either excess or deficient volatility may be possible.

### B.3 Event Study Implications

Many recent studies have investigated abnormal average return performance or 'drift' following public news arrival. As mentioned in the introduction, a striking regularity in virtually all these studies is that average post-event abnormal price trends are of the same sign as the average initial event-date reaction. We now slightly generalize the model to address this event-based return predictability.

Sophisticated managers or analysts who are not overconfident are likely to undertake certain visible actions, such as repurchasing shares or making buy recommendations, selectively when a firm's shares are undervalued by the market. We will show that the nature of the stock price reaction to an event depends critically on whether or not the event is related to the date 2 mispricing by the market.

We assume that the date 2 signal is no longer public, but is instead received *privately* by the firm's manager (or other individual such as an analyst), and that this individual takes an action (the 'event') which is publicly observed and fully reveals the signal. Let $P_2^C(s_2)$ be the valuation that would be placed on the security by an overconfident investor at date 2 were he to observe the signal $s_2$ in addition to his signal $s_1$. (Since we examine events that fully reveal $s_2$, this is in equilibrium just the post-event stock price $P_2$.) Let $P_2^R(s_2)$ be the comparable valuation that would be set by a fully rational investor. The date 2 *mispricing* then is defined as the difference $P_2^R(s_2) - P_2^C(s_2)$. We define different kinds of events as follows.

**Definition 1** *An* **event** *is a random variable that depends only on the information signals $s_1$ and $s_2$. A* **non-selective event** *is an event that is independent of the date 2 mispricing $P_2^R(s_2) - P_2^C(s_2)$. A* **selective event** *is an event whose occurrence and/or magnitude depends on the date 2 mispricing.*

A simple type of non-selective event is a random variable that is linearly related only to the second signal $s_2$.

**Proposition 4** *If overconfident investors observe a non-selective event:*

1. *The true expected post-announcement abnormal price change is zero.*
2. *Conditional on the pre-event return, the covariance between the announcement-date and the post-announcement price change is positive, i.e.,* $\text{cov}(P_3 - P_2, s_2|P_1 - P_0) > 0$.

Since a non-selective event is an action that is unrelated to the pricing error at date 2, it tells us nothing about mean future price movements. Although the market underreacts to the event, it is equally likely to be underreacting downward as upward. Part 1 therefore indicates that there will be no systematic post-announcement drift following events that are unrelated to the prior market mispricing. Thus, Proposition 4 refutes the conventional interpretation of drift as being equivalent to underreaction to new information.

The lack of event-based predictive power for future returns is surprising given the positive autocorrelation of event-date and post-event price changes (Proposition 1). However, even though the event is unrelated to the prior mispricing, the more underpriced the security, the more positive on average will be the stock price reaction to further news. Thus, a favorable event-date stock price change is associated with a positive future average trend. Clearly, then, even though the event itself does not predict future returns, the market is inefficient.

Part 2 of Proposition 4 predicts larger post-event average returns the more the non-selective event (perhaps a cash flow surprise) and the pre-event stock price runup are in opposition (e.g., positive pre-event runup and negative event).[8] Intuitively, holding constant the private signal (as reflected in $P_1$), the higher is the public signal, the more likely that the fundamental $\theta$ is high, and therefore the bigger the average shortfall of the private signal relative to the fundamental. Thus, a higher public signal is associated with a larger (more positive) post-event return.

Both Parts 1 and 2 of Proposition 4 can be tested using data on specific non-selective events. These are presumably events that are not initiated by an informed party such as a manager with an incentive to take into account mispricing. Such events might include news about product demand that emanates from outside of the company (e.g., news about competitors' actions), or regulatory and legislative outcomes (e.g., FDA decisions on drugs proposed by a pharmaceutical company).

We now show that selective public events, i.e., events that are correlated with pre-event stock mispricing, will forecast future price changes. Consider a manager who observes $P_1$ (and therefore infers the private signal $s_1$) and receives his own signal $s_2$ at date 2. The manager can undertake a debt/equity exchange offering, and the attractiveness of a larger exchange depends on how high the market price is relative to fundamental value. He can condition the size of the offering on the mispricing at date 2, which he knows precisely, since he knows both $s_1$ and $s_2$. It can easily be shown that in this setting the date 2 pricing error is proportional to the expected error in the private signal, $\epsilon^* \equiv E[\epsilon|P_1, s_2]$, where the expectation is again taken with respect to rational beliefs. For tractability, we consider selective events that are linear functions of the date 2 mispricing.

When $\epsilon^* < 0$, the manager believes the market has undervalued the firm, so the firm can 'profit' by exchanging debt for equity; the more undervalued the firm, the greater the size of the offering. If $\epsilon^* > 0$, an equity-for-debt swap would be preferred instead. It is easy to show that

$$E[P_3 - P_2|\epsilon^* > 0] < 0 < E[P_3 - P_2|\epsilon^* < 0], \tag{8}$$

i.e., events taken in response to market undervaluation (e.g., repurchase) are associated with high post-event returns, and events taken in response to overvaluation (e.g., new issue) with low post-event returns.

**Proposition 5** *If investors are overconfident, then selective events that are initiated when the stock is undervalued (overvalued) by the market will on average be associated with positive (negative) announcement-date abnormal price changes and will on average be followed by positive (negative) post-announcement abnormal price changes.*

In Proposition 4 there was underreaction to news arrival but no drift. Here, drift results from the combination of underreaction and event selection based on market mispricing. Thus, the model offers the new empirical implication that the phenomenon of abnormal post-event drift will be concentrated in events that select for market mispricing. Evidence recently brought to our attention supports this implication: Cornett, Mehran, and Tehranian (1998) find that 'involuntary' issues undertaken by banks to meet capital requirements are not associated with post-event drift, whereas 'voluntary' bank issues are associated with negative post-event abnormal performance. Since involuntary issues are likely to be less selective than voluntary ones, this evidence is consistent with the model.

If the announcement of an upcoming Initial Public Offering (IPO), like an Seasoned Equity Offering (SEO) announcement, reflects managers' 'bad news', then Proposition 5 implies long run underperformance following IPOs as well. Since IPO firms are private prior to the event, we have no data on the announcement-date reaction to an upcoming IPO. However, the consistent findings of negative stock price reactions to seasoned equity issue announcements, and of inferior post-IPO accounting performance (Jain and Kini (1994), Mikkelson, Partch, and Shah (1997), Teoh, Wong, and Rao (1997), Loughran and Ritter (1997)), suggest that an IPO announcement is indeed on average bad news.[9] If so, the evidence that IPOs internationally exhibit long-run average underperformance for several years after the issue (Ritter (1991) and Loughran, Ritter, and Rydqvist (1994)) is consistent with the model.

The event-based return predictability of Proposition 5 is not equivalent to 'underreaction' to corporate events. Underreaction to public signals (as implied by overconfidence) induces positive autocorrelation of returns at the event date. However, the event realization (in contrast to the event-date return) may not predict future abnormal returns unless event size/occurrence is correlated with prior market mispricing.

We have interpreted the model in terms of firms buying or selling shares to profit from mispricing. An alternative interpretation is that a manager with favorable information ($\epsilon^* < 0$) would like to *signal* good news to the market, and chooses an action (such as a repurchase, dividend, debt for equity swap, or stock split) to reveal his information. With a continuous signal, such behavior typically leads to full revelation, consistent with our assumption that $\epsilon^*$ is revealed to the market at the event date.[10]

Whether the model of this section is consistent with the well-known phenomenon of post-earnings announcement 'drift' depends on whether earnings announcements are selective events. An earnings report is favorably selective if managers report higher earnings, *ceteris paribus*, when the market undervalues their firm. A manager has an incentive to do so if he is averse to low levels of short-term stock price or personal reputation.[11] Further, managers have a great deal of discretion over earnings levels both through accounting adjustments (accruals), and by shifting the timing of actual cash flows. Accounting adjustments seem to reflect managers' inside information, as evidenced by the announcement effect of accruals on returns (distinct from the effect of cash flows); see Wilson (1986). There is extensive evidence that managers use their accounting discretion strategically to achieve their goals, such as meeting loan covenant requirements, winning proxy fights, obtaining earnings-based bonuses, and avoiding taxes; Teoh, Wong, and Rao (1997) reference about 30 such studies. If managers adjust earnings selectively, Proposition 5 can account for post-earnings drift. The dynamic confidence setting of Section III provides a distinct explanation for post-earnings announcement drift that obtains even if earnings are non-selective.

Since the date 1 expected value of $\epsilon^*$ is perfectly positively correlated with $P_1$ (they both are linearly increasing functions of $s_1$), variables such as market/book or runup $(P_1 - \bar{\theta})$ are potential measures of mispricing. As we have assumed that the size of a selective event depends on the size of the misvaluation, it follows that the size and sign of the selective event varies with the measures of mispricing. We therefore have:

**Proposition 6** *1. The expected size of a positive (negative) selective event is increasing (decreasing) in measures of the firm's mispricing.*

*2. The probability that a positive (negative) selective event will occur increases (decreases) with measures of the firm's mispricing.*

We tentatively identify mispricing with variables that contain market price such as market/book ratios. The analysis then predicts that repurchases and other favorable events will tend to occur when market, industry, or firm market/book or price/earnings ratios are low, and equity issuance and other adverse selective events when such ratios are high. This is consistent with evidence that the frequency of IPOs is positively related to the market/book ratio in the company's industrial sectors (Pagano, Panetta, and Zingales (1998)), and that in many countries the value and number of IPOs is positively associated with stock market levels (Loughran, Ritter, and Rydqvist (1994), Rees (1996), Ljungqvist (1997)).

The analysis also implies that event date price changes (for a given type of event) should be positively correlated with post-announcement returns. This is just underreaction, and follows under the conditions of Proposition 1.[12] Also, in the model, because the pre-event price runup maps one-to-one with market mispricing, better pre-event price performance is associated with worse post-event performance (either including or excluding the event date). This follows because $\mathrm{cov}(P_3 - P_2, P_1 - P_0) < 0$ and $\mathrm{cov}(P_3 - P_1, P_1 - P_0) < 0$. Intuitively, mispricing arises from overreaction to private information, firms select events based on mispricing, and this causes post-event returns to be related to pre-event returns. However, the latter implication is not robust to reasonable generalization of the assumptions to allow for the possibility that public information can arrive at date 0 or 1.

Consider, for example, the case of dividend announcements. Firms that have been performing well enough to generate a lot of cash are more likely to boost dividends. Thus, a dividend increase will be associated not only with market undervaluation at date 2 (unfavorable date 1 *private* signal), but also with good past performance (favorable date 0 or 1 *public* signal). In this scenario, while the event-date and post-event mean abnormal returns are both positive, the sign of the pre-event mean return will be ambiguous. We have verified formally that if the event choice (dividend) increases with both a past (date 1) public signal and the degree of market undervaluation, then the event may be associated with a positive average runup, a positive average event date return, and a positive average post-event return.[13]

More generally, whether prior runup (or other price-related indicators such the fundamental/price ratios) is a measure of mispricing depends on whether the event in question is mainly selective for mispricing, or depends heavily on past fundamental public performance measures (such as past earnings). Many events, such as dividends and stock splits, may be selective owing to a signalling motive. But events in which the firm trades against the market, such as exchange offers, repurchases, and new issues, provide an incentive to earn a trading profit. This provides an incentive to be selective above and beyond any signalling motive. Thus, runup and price/fundamental ratios should be better measures of mispricing for such market-exploitation events than for pure signalling events.

### B.4 Empirical Implications

The model provides the following implications, which are either untested or have been tested only on a subset of possible events:

1. Average post-event returns of the same sign as average event-date returns for selective events, and zero post-event drift for non-selective events;

2. A positive correlation between initial event-date stock price reactions and post-event performance for public events;

3. A positive correlation between the size of a selective event (e.g., a repurchase or the announcement of a toehold stake) and post-event return, but no such correlation for non-selective events (*e.g.*, news disclosed by outside sources, especially if macroeconomic or industry-wide, such as news about product demand or input prices, production processes, and regulatory events);

4. Larger post-event average returns the more the nonselective event and the pre-event stock price runup are in opposition;

5. Greater average long-term reversal of price moves occurring on dates when there are no public news events about a firm reported in public news media than price moves occurring on public event dates;

6. Greater selective event sizes (e.g., greater repurchases) when mispricing measures (e.g., price/fundamental ratios or past runup) are high; and,

7. Greater probability of a good-news (bad news) selective event when the security is more underpriced (overpriced).

The overconfidence theory has further implications for managerial policy related to implications (6) and (7) above. We expect firms to issue securities when they believe their stocks are overvalued. If investors are overconfident, such overvaluation may be measured by recent increases in firm, industry or aggregate stock market prices, or with high price/fundamental ratios. Conversely, firms should repurchase after rundowns when the market appears to undervalue the firm. Thus, if managers act to exploit mispricing, there will be both general and industry-specific financing and repurchase booms.

The theory also suggests that when the market market undervalues the firm, there should be a tilt away from dividends toward repurchase. Further, when a stock is underpriced (perhaps after rundowns or when firm or aggregate market/book ratios are low), the firm, acting in current shareholders' interests should, *ceteris paribus*, favor rights over public issues. Similarly, the firm should tilt toward debt rather than equity issues to avoid diluting current shareholders. Thus, the theory offers a possible solution to what Opler and Titman (1996) call a major puzzle from the perspective of optimal capital structure theory, that after a rise in market prices, firms tend to issue more equity rather than debt.[14]

Since these predictions seem quite intuitive, it is easy to forget that the directions would reverse in alternative models of market mispricing. For example, in a setting where the market always underreacts, firms with high recent runups or low fundamental/price ratios will, *ceteris paribus*, tend to be *undervalued*, so that (inconsistent with the evidence) we would observe repurchases rather than equity issues in such situations.

# III. Outcome-Dependent Confidence

The implications described so far are based on a fixed confidence level. However, psychological evidence and theory suggest that actions and resulting outcomes affect confidence; events that confirm an individual's beliefs and actions tend to boost confidence too much, while disconfirming events weaken confidence too little (see Section I). Taking into account this psychological pattern leads to implications similar to those in the static section, except that there is also short-run momentum in stock prices and event-based predictability even for non-selective events.

Consider an informed individual who initially is not overconfident, and who buys or sells a security based on his private information. A public signal *confirms* his trade if they have the same sign ("buy" and a positive signal, or "sell" and a negative signal). We assume that if the later public signal confirms his trade, the individual becomes more confident, and if it disconfirms his confidence decreases by little or remains constant. This implies that *on average*, public information can increases confidence, intensifying overreaction. The continuing overreaction leads to positive autocorrelation during the initial overreaction phase. As repeated public information arrival draws the price back toward fundamentals, the initial overreaction is gradually reversed in the long run.

The above process yields a hump-shaped impulse response function for a private signal as illustrated by the dashed lines in Figure 1. (The date 0/1 line overlaps the solid lines showing the impulse response for the static model.) The Figure shows two possible date 1 prices, and the paths for expected price conditional on the date 1 move. It can be seen that with outcome-dependent confidence, there are smooth overreaction and correction phases. Pairs of returns drawn from these phases will be positively correlated whereas the pair which straddles the extremum will be negatively correlated. The overall autocorrelation involving contiguous price changes will be positive if the extremum-straddling negative correlation is sufficiently small. However, price changes that are separated by long lags are likely to straddle the extremum of the impulse-response function, and will therefore exhibit negative autocorrelations. Thus, the pattern of momentum at short lags and reversal at long lags arises naturally from the model.

We present two models with dynamic confidence that capture this intuition. The model presented in Subsection A is tractable but highly stylized. The model presented in Subsection B allows us to develop more complex implications, but can only be solved by simulation.

## A. The Simple Model with Outcome Dependent Confidence

We modify the basic model of Section II as follows. We still allow for, but no longer require, initial overconfidence, so $\sigma_C^2 \leq \sigma_\epsilon^2$. For tractability, the public signal is now discrete, with $s_2 = 1$ or $-1$ released at date 2. We assume that the precision assessed by the investors at date 2 about their earlier private signal depends on the realization of the public signal in the following way. If

$$\text{sign}(\theta + \epsilon) = \text{sign}(s_2), \tag{9}$$

confidence increases, so investors' assessment of noise variance decreases to $\sigma_C^2 - k$, $0 < k < \sigma_C^2$. If

$$\mathrm{sign}(\theta + \epsilon) \neq \mathrm{sign}(s_2), \tag{10}$$

confidence remains constant, so noise variance is still believed to be $\sigma_C^2$.

The probability of receiving a public signal $+1$ is denoted by $p$. For a high value to be a favorable indicator of value, $p$ must tend to increase with $\theta$. However, allowing $p$ to vary with $\theta$ creates intractable non-normalities. We therefore examine the limiting case where the signal is virtually pure noise, so that $p$ is a constant. (Appendix C provides a discrete model which derives similar results using an informative public signal.)

Given normality of all random variables, the date 1 price is

$$P_1 = E_C[\theta | \theta + \epsilon] = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_C^2}(\theta + \epsilon). \tag{11}$$

The date 0 price $P_0 = 0$, the prior mean. If $\mathrm{sign}(\theta + \epsilon) \neq \mathrm{sign}(s_2)$, then confidence is constant. Since the public signal is virtually uninformative, the price (virtually) does not move at date 2. However, if $\mathrm{sign}(\theta + \epsilon) = \mathrm{sign}(s_2)$, then the new price is calculated using the new level of the assessed variance of $\epsilon$. This price, denoted by $P_{2C}$, is

$$P_{2C} = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_C^2 - k}(\theta + \epsilon). \tag{12}$$

### A.1 Implications of the Simple Model

Explicit calculations and expressions for covariances for this subsection are in Appendix D. It can easily be shown that

$$\mathrm{cov}(P_2 - P_1, P_1 - P_0) > 0. \tag{13}$$

Thus, the model shows that the overreaction phase, not just the correction phase, can contribute positively to short-term momentum. As a result,

$$\mathrm{cov}(P_3 - P_1, P_1 - P_0) \;<\; 0; \tag{14}$$

$$\mathrm{cov}(P_3 - P_2, P_2 - P_1) \;<\; 0, \tag{15}$$

because the dates 1 and 2 overreactions must be reversed in the long-term.

Intuitively, further dates of noisy public information arrival should eventually cause the mispricing to be corrected (so long as confidence does not explode infinitely). This process causes positive autocorrelation during the correction phase, just as in the basic model of Section II. To examine this, let us add a date $3'$ between dates 2 and 3, where a public signal $\theta + \eta$ is released. For simplicity, we assume that overconfidence is not affected by the release of the second public signal. As in Section II, $\eta$ is a zero mean, normally distributed variable with variance $\sigma_p^2$, and is independent of all other random variables. The price at date $3'$ when overconfidence is not revised at date 2 is given by equation (6). When overconfidence is revised at date 2, the price at date $3'$, denoted by $P_{3'C}$ is given by the same expression as equation (6), except that $\sigma_C^2$ is replaced by $\sigma_C^2 - k$; i.e.,

$$P_{3'C} = \frac{\sigma_\theta^2(\sigma_C^2 - k + \sigma_p^2)}{D}\theta + \frac{\sigma_\theta^2\sigma_p^2}{D}\epsilon + \frac{\sigma_\theta^2(\sigma_C^2 - k)}{D}\eta, \tag{16}$$

where $D \equiv \sigma_\theta^2(\sigma_C^2 - k + \sigma_p^2) + (\sigma_C^2 - k)\sigma_p^2$.

With the extra date added to the model, it is easy to show that all of the remaining single-period-price change autocorrelations are negative except for $\text{cov}(P_3 - P_{3'}, P_{3'} - P_2)$, which is positive. This can be explained as follows. Date 2 is the extremum of the impulse response function (the 'hump' or 'trough' date after which the average correction begins). By equation (A65) and the above, the single-period-price change single-lag autocorrelations that fall entirely within either the overreaction phase or within the correction phase are positive, while the single-period-price-change single-lag autocorrelation that straddles the extremum is negative.[15]

Under appropriate parameter assumptions, the negative single-lag autocorrelation surrounding the extremum will be arbitrarily close to zero. This will occur if either the extra overreaction or the start of the correction is weak (or both). The extra overreaction is small if confidence is boosted only slightly ($k > 0$ small) when an investor's trade is confirmed by public news. The initial correction is slight if the further noisy public signal is not very informative ($\sigma_\eta^2$ large). When parameter values are such that this straddling autocorrelation is not too large, it will be outweighed by the positive autocorrelations during the hearts of the overreaction or correction phases. In other words, an econometrician calculating autocorrelations unconditionally would find, in a large sample, a positive single-lag autocorrelation. In contrast, longer-lag pairs of price changes that straddle the extremum of the impulse response function will tend to be opposed, because a price change drawn from the overreaction phase tends to be negatively correlated with a price change drawn from the correction phase. Thus, the overconfidence theory provides a joint explanation for both short-term momentum and long-term reversals.

**Proposition 7** *If investor confidence changes owing to biased self-attribution, and if overreaction or correction is sufficiently gradual, then stock price changes will exhibit unconditional short-lag positive autocorrelation ('momentum') and long-lag negative autocorrelation ('reversal').*

According to Jegadeesh and Titman (1993), their momentum evidence is "... consistent with delayed price reactions to firm-specific information." Proposition 7 offers a very different possible interpretation, namely, that momentum occurs not because the market is slow to react to news, but because the market initially overreacts to the news, and later public news triggers further overreaction to the initial private signal. More generally, Proposition 7 refutes the common casual equating of positive versus negative autocorrelations with underreaction versus overreaction to new information. While negative autocorrelations result from overreaction in the model, positive autocorrelations also result from continuing overreaction (followed by underreaction in the correction of this error).

Evidence from the psychological literature suggests that individuals tend to be more overconfident in settings where feedback on their information or decisions is slow or inconclusive than where the feedback is clear and rapid (Einhorn (1980)). Thus, mispricing should be stronger in stocks which require more judgment to evaluate, and where the feedback on this judgment is ambiguous in the short run, such as for growth stocks whose value is, on average, more strongly tied to hard to value growth options. This conjecture is consistent with recent work by Daniel and Titman (1998), which finds that the momentum effect is strong in growth stocks, but is weak or non-existent in value stocks. This line of reasoning also suggests that momentum should be stronger for stocks that are difficult to value, such as those with high R&D expenditures or intangible assets.

## B. A Dynamic Model of Outcome-Dependent Confidence

We now extend this model to an arbitrary number of periods and present numerical simulations. The analysis implies patterns of security price-change autocorrelations consistent with the findings of Subsection A. It also yields further implications for the correlation between public information announcements (such as managers' forecasts or financial reports of sales, cash flows or earnings) and future price changes.

### B.1 The Model

We retain the basic structure considered in earlier sections. We assume that the investor has a prior on the precision of his private signal, and uses an updating rule that reflects self-attribution bias. As before, the (unobservable) value of a share of the firm's stock is $\tilde{\theta} \sim \mathcal{N}(0, \sigma_\theta^2)$. The public noise variance $\sigma_\theta^2$ is common knowledge. At date 1, each informed investor receives a private signal $\tilde{s}_1 = \tilde{\theta} + \tilde{\epsilon}$ where $\tilde{\epsilon} \sim \mathcal{N}(0, \sigma_\epsilon^2)$. At dates 2 through $T$, a public signal $\tilde{\phi}_t$ is released, $\tilde{\phi}_t = \tilde{\theta} + \tilde{\eta}_t$, where $\tilde{\eta}_t$ is $i.i.d$ and $\tilde{\eta}_t \sim \mathcal{N}(0, \sigma_\eta^2)$. The variance of the noise, $\sigma_\eta^2$, is also common knowledge. Let $\Phi_t$ be the average of all public signals through time $t$:

$$\Phi_t = \frac{1}{(t-1)} \sum_{\tau=2}^{t} \tilde{\phi}_\tau = \theta + \frac{1}{(t-1)} \sum_{\tau=2}^{t} \tilde{\eta}_\tau. \tag{17}$$

The average public signal $\Phi_t$ is a sufficient statistic for the $t-1$ public signals, and $\tilde{\Phi}_t \sim \mathcal{N}(\theta, \sigma_\eta^2/(t-1))$.

As before, an informed investor forms expectations about value rationally (using Bayesian updating) except for his perceptions of his private information precision. The error variance $\sigma_\epsilon^2$ is incorrectly perceived by the investor. He estimates $\sigma_\epsilon^2$ using an *ad hoc* rule described below. At time 1, the investor believes that the precision of his signal, $v_{C,1} = 1/\sigma_{C,1}^2$, is greater than the true precision $v_\epsilon = 1/\sigma_\epsilon^2$. At every subsequent release of public information the investor updates his estimate of the noise variance. If the new public signal ($\phi_t$) confirms the investor's private signal $s_1$, and the private signal is not too far away from the public signal, then the investor becomes more confident in his private signal. If the new public signal disconfirms his private signal, the investor revises the estimated precision downwards, but not by as much. Thus, the specific updating rule that we implement is:

$$\text{if} \begin{cases} sign(s_1 - \Phi_{t-1}) = sign(\phi_t - \Phi_{t-1}) \text{ and } |s_1 - \Phi_{t-1}| < 2\sigma_{\Phi,t} & \text{then } v_{C,t} = (1+\overline{k})v_{C,t-1} \\ \text{otherwise} & v_{C,t} = (1-\underline{k})v_{C,t-1}, \end{cases} \tag{18}$$

where $\sigma_{\Phi,t}$ is the standard deviation of $\Phi$ at time $t$. We impose the restriction that $\overline{k} > \underline{k} > 0$. The ratio $(1+\overline{k})/(1-\underline{k})$ is an index of the investor's attribution bias.[16,17]

### B.2 The Equilibrium

Since the investor is risk-neutral and the risk-free rate is zero, at each point in time the stock price is the expectation of its terminal value:

$$P_t = E_C[\tilde{\theta}|s_1, \phi_2, \ldots, \phi_t] = E_C[\tilde{\theta}|s_1, \Phi_t]. \tag{19}$$

Define $v_\theta = 1/\sigma_\theta^2$, and $v_\eta = 1/\sigma_\eta^2$. The price of the security at time $t$ is given by:

$$\tilde{P}_t = E_C[\tilde{\theta}|s_1, \Phi_t] = \frac{(t-1)v_\eta \Phi_t + v_{C,t} s_1}{v_\theta + v_\eta + v_{C,t}}. \tag{20}$$

Recall that the precision of $\Phi_t$ is $(t-1)v_\eta$.

### B.3 Simulation Results and Empirical Implications

For the simulation we use the parameters $\overline{k} = 0.75$, $\underline{k} = 0.1$, $\sigma_\theta^2 = \sigma_\epsilon^2 = 1$, and $\sigma_\eta^2 = 7.5$. We also make the investor's initial estimate of his precision equal to the true precision of his private signal. We perform this simulation 50,000 times, each time redrawing the value $\theta$, the private signal $s_1 = \theta + \epsilon$, and the public information set $\phi_t$, for $t = 2, \ldots T$.

It is useful to first illustrate the dynamic price path implied by the model for specific realizations of $s_1$ and $\theta$. Figure 2 shows the average price path following a private-signal of $s_1 = 1$ when $\theta = 0$, so that the informed investors' signal is unduly favorable. The price initially jumps from 0 up to 0.5, a rational assessment. On average, the price continues moving up, reaching a maximum of 0.7366 in period 16. The average price then declines, and eventually asymptotes to zero. Thus, there is an initial overreaction phase in which the price moves away from the true value as the investor's attribution bias causes him to place more weight, on average, on his private information. Eventually the public information become precise enough that the investor revises his valuation of the security downward. This is the correction phase. A similar hump-shaped pattern holds for an investors' self-perceived precision (confidence) as a function of time. This changing confidence is the source of the overreacting average price trend.

Figure 3 presents the *unconditional* average autocorrelations (at lags between 1 period and 119 periods), where now $\tilde{\theta}$ and $\tilde{s}_1$ are resampled for each iteration. This figure confirms the intuition derived from Figure 2 that short-lag price change autocorrelations should be positive and long-lag autocorrelations should be negative.

Several papers examine 'long-horizon' regressions of long period returns on past returns (see, e.g., Fama and French (1988)) rather than long-lag autocorrelations of short-period returns. In our model, it is straightforward to show that there is a one-to-one mapping between price change autocorrelations and more standard test statistics such as variance ratios or long-horizon regression coefficients. In unreported simulations, these coefficients exhibit behavior similar to that of the autocorrelations. Short horizon regression coefficients are positive and long-horizon ones are negative, consistent with empirical literature on momentum and reversals.

The conclusions of this simulation are summarized as follows.

**Result 1** *In the biased self-attribution setting of Subsection B, if the true share value $\theta = 0$ and the initial private signal $s_1 = 1$, then with sufficient attribution bias the average price at first rises and then gradually declines. This contrasts with a steadily declining price path if there is no attribution bias. In the biased self-attribution setting, average self-perceived precision also initially rises and then declines.*

**Result 2** *In the biased self-attribution setting of Subsection B, short-lag autocorrelations (correlating single-period price changes with single-period price changes) are positive and long-lag autocorrelations are negative.*

**Result 3** *In the biased self-attribution setting of Subsection B, short-term autocorrelations* *are positive and long-horizon autocorrelations are negative.*

Recent research indicates strong and consistent evidence of momentum in the US and European countries, but weak and insignificant evidence of momentum in Japan (see, for example, Haugen and Baker (1996) and Daniel, Titman, and Wei (1996)). There is corresponding evidence of a difference in biased self-attributions in Western versus Asian groups, especially Japan. For example, Kitayama, Takagi, and Matsumoto (1995) review 23 studies conducted in Japan which find essentially no evidence of self-enhancing biases in attribution. These findings suggest the more general prediction that cultures in which there is little or no self-enhancing attribution bias (e.g., other Asian countries such as Korea, PRC, and Taiwan; see the references in Kitayama, Takagi, and Matsumoto (1995)) should have weak momentum effects.

De Long, Shleifer, Summers, and Waldmann (1990a) have derived security return autocorrelations in a model with mechanistic positive feedback traders. Our approach differs in explicitly modeling the decisions of quasi-rational individuals. Our model provides one possible psychological foundation for a stochastic tendency for trades to be correlated with past price movements, which can create an appearance of positive feedback trading.

### B.4 Correlation of Accounting Performance with Subsequent Price Changes

Finally, we consider the implications of this model for the correlation between accounting performance and future price changes. Accounting information (sales, earnings, etc.) can be thought of as noisy public signals about $\theta$, so in this subsection we interpret the $\phi$s as accounting performance change measures. Consider the first public signal (at $t = 2$). If this is positive, the first private signal was probably also positive. Based on the momentum results in this section, this suggests that prices will continue to increase after the arrival date of the public signal, consistent with empirical evidence on earnings-based return predictability. Eventually prices will decline as the cumulative public signal becomes more precise and informed investors put less weight on their signal. Thus, the analysis of this section suggests that earnings-based return predictability, like stock-price momentum, may be a phenomenon of continuing overreaction.[18] In the long-run, of course, the security price will return to its full-information value, implying long-run negative correlations between accounting performance and future price changes. This conjecture is consistent with the empirical evidence discussed in Appendix A, though, from an empirical standpoint, statistical power to detect long-lag autocorrelations is limited.

To evaluate the above conjecture, we again calculate average correlations using our simulation as follows. For each $\tilde{\phi}_t$ (for $t = 2, 120$) we calculate the 'earnings' surprise, defined as:

$$\Delta e_t = \tilde{\phi}_t - \Phi_t = \tilde{\phi}_t - E[\tilde{\phi}_t | \phi_2, \phi_3, \ldots, \phi_{t-1}], \tag{21}$$

the deviation of $\phi_t$ from its expected value based on all past public signals. Then, we calculate the set of sample correlations between the $\Delta e_t$ and price changes $\tau$ periods in the future $\Delta P_{t+\tau} = P_{t+\tau} - P_{t+\tau-1}$. These correlations are then averaged over the Monte Carlo draws. The average correlations are plotted in Figure 4. This simulation yields:

**Result 4** *In the biased self-attribution setting of Subsection B, short-lag correlations between single-period stock price changes and past earnings are positive, and long-lag correlations can be positive or negative.*

To summarize, the analysis suggests that the conclusion from the basic model that investors overreact to private signals holds in the dynamic model. While investors underreact on average to public signals, public signals initially tend to stimulate additional overreaction to a previous private signal. Thus, underreaction is mixed with continuing overreaction.

In the model of this section, earnings-based return predictability and momentum both arise from self-attribution bias. Further, the literature cited in Subsection B.3 suggests that the magnitude of this bias varies systematically across countries. Based on these observations, the self-attribution model suggests a positive relationship across international markets between the strength of the momentum effect and that of the post-earnings announcement drift.

# IV. Conclusion

Empirical securities markets research in the last three decades has presented a body of evidence with systematic patterns that are not easy to explain with rational asset pricing models. Some studies conclude that the market underreacts to information, while others find evidence of overreaction. We have lacked a theory to integrate this evidence, and to make predictions about when over- or underreaction will occur.

This paper develops a theory based on investor overconfidence and on changes in confidence resulting from biased self-attribution of investment outcomes. The theory implies that investors will overreact to private information signals and underreact to public information signals. In contrast with the common correspondence of positive (negative) return autocorrelations with underreaction (overreaction) to new information, we show that positive return autocorrelations can be a result of continuing overreaction. This is followed by by long-run correction. Thus, short-run positive autocorrelations can be consistent with long-run negative autocorrelations.

The theory also offers an explanation for the phenomenon of average public event stock price reactions of the same sign as post-event long-run abnormal returns. This pattern has sometimes been interpreted as market underreaction to the event. We show that underreaction to new public information is neither a necessary nor a sufficient condition for such event-based predictability. Such predictability can arise from underreaction only if the event is chosen in response to market mispricing. Alternatively, predictability can arise when the public event triggers a continuing overreaction. For example, post-earnings announcement drift may be a continuing overreaction triggered by the earnings announcement to pre-event information.

The basic noise trading approach to securities markets (e.g., Grossman and Stiglitz (1980), Shiller (1984), Kyle (1985), Glosten and Milgrom (1985), Black (1986), De Long, Shleifer, Summers, and Waldmann (1990b), and Campbell and Kyle (1993)) posits that there is variability in prices arising from unpredictable trading that seems unrelated to valid information. Our approach is based on the premise that an important class of mistakes by investors involves the misinterpretation of *genuine* new private information. Thus, our model endogenously generates trading mistakes that are correlated with fundamentals. Modeling the decision problems of quasi-rational traders imposes restrictions on trade distributions which are not obvious if distributions are imposed exogenously. This structure provides predictions about the dynamic behavior of asset prices which depend on the particular cognitive error that is assumed. For example, *under*confidence also gives rise to quasi-rational trading that is correlated with fundamentals, but gives rise to empirical predictions which are the reverse of what the empirical literature finds. Specifically, if informed investors are underconfident ($\sigma_C^2 > \sigma_\epsilon^2$), there will be insufficient volatility relative to the rational level, long-run return continuation, and negative correlation between selective events such as repurchase and post-event returns. Of course, one could arbitrarily specify whatever pattern of correlated noise is needed to match empirically observed *ex post* price patterns. Such an exercise would merely be a relabeling of the puzzle, not a theory. Instead, we examine a form of irrationality consistent with well-documented psychological biases, and our key contribution is to show that these biases induce several of the anomalous price patterns documented in the empirical literature.

Some models of exogenous noise trades (e.g., De Long, Shleifer, Summers, and Waldmann (1990b), Campbell and Kyle (1993)) also imply long-run reversals and excess volatility because of the time-varying risk premia induced by these trades. Our approach additionally reconciles long-run reversals with short-term momentum, explains event-based return predictability, and offers several other distinct empirical predictions (see Subsections II.B.1 through II.B.3).

As noted in the introduction, a possible objection to models with imperfectly rational traders is that wealth may shift from foolish to rational traders until price-setting is dominated by rational traders. For example, in our model the overconfident informed traders lose money on average. This outcome is similar to the standard result that informed investors cannot profit from trading with uninformed investors unless there is some 'noise' or 'supply shock.' However, recent literature has shown that in the long-run rational traders may not predominate. DeLong, Shleifer, Summers, and Waldman (1990b, 1991) point out that if traders are risk-averse, a trader who underestimates risk will allocate more wealth to risky, high expected return assets. If risk averse traders are overconfident about genuine information signals (as in our model), overconfidence allows them to exploit information more effectively. Thus, the expected profits of the overconfident can be greater than those of the fully rational (see Daniel, Hirshleifer, and Subrahmanyam (1998))

Furthermore, owing to biased self-attribution, those who acquire wealth through successful investment may *become* more overconfident (see also Gervais and Odean (1998)). Another distinct benefit of overconfidence is that this can act like a commitment to trade aggressively. Since this may intimidate competing informed traders, those known to be overconfident may earn higher returns (see Kyle and Wang (1997) and Benos (1998)).

Recent evidence suggests that event-based return predictability varies across stocks (*e.g.*, Brav and Gompers (1997)). Moving beyond the confines of the formal model, we expect the effects of overconfidence to be more severe in less liquid securities and assets. Suppose that all investors are risk averse and that prices are not fully-revealing (perhaps because of noisy liquidity trading). If rational arbitrageurs face fixed setup costs of learning about a stock, then large liquid stocks will tend to be better arbitraged (more rationally priced) than small stocks, because it is easier to cover the fixed investigation cost in large, liquid stocks. This suggests greater inefficiencies for small stocks than for large stocks, and for less liquid securities and assets such as real estate than for stocks. Furthermore, since the model is based on overconfidence about private information, the model predicts that return predictability will be be strongest in firms with the greatest information asymmetries. This also implies greater inefficiencies in the stock prices of small companies. Furthermore, proxies for information asymmetry such as the adverse selection component of the bid-ask spread should also be positively related to momentum, reversal, and post-event drift.

It is an open question whether the overconfident traders in the model can be identified with a specific category of investor, such as institutions, other investment professionals, small individual investors, or all three. Even small individual investors, who presumably have less information, may still be overconfident. The uninformed investors of the model could be interpreted as being contrarian-strategy investors (whether institutions or individuals).

(Some smart contrarian investors could be viewed as rational and informed; including such traders would not change the qualitative nature of the model predictions.) An identification of the confidence characteristics of different observable investor categories may generate further empirical implications, and is an avenue for further research.

## Appendix A: Securities Price Patterns

This appendix cites the relevant literature for the anomalies mentioned in the first paragraph of the introduction. Out-of-sample tests (in time and location) have established several of these patterns as regularities.

**Underreaction to Public News Events (event-date average stock returns of the same sign as average subsequent long-run abnormal performance):** Events for which this has been found include:

1. Stock splits (Grinblatt, Masulis, and Titman (1984), Desai and Jain (1997), and Ikenberry, Rankine, and Stice (1996))

2. Tender offer and open market repurchases (Lakonishok and Vermaelen (1990), Ikenberry, Lakonishok, and Vermaelen (1995))

3. Analyst recommendations (Groth, Lewellen, Scharbaum, and Lease (1979), Bjerring, Lakonishok, and Vermaelen (1983), Elton, Gruber, and Gultekin (1984), Womack (1996), and Michaely and Womack (1996)),

4. Dividend initiations and omissions (Michaely, Thaler, and Womack (1995))

5. Seasoned issues of common stock (Loughran and Ritter (1995), Spiess and Affleck-Graves (1995), Teoh, Welch, and Wong (1998), but see the differing evidence for Japan of Kang, Kim, and Stulz (1996))

6. Earnings surprises (at least for a period after the event) (Bernard and Thomas (1989, 1990), Brown and Pope (1996))

7. Public announcement of previous insider trades (Seyhun (1997); see also Seyhun (1986), Seyhun (1988) and Rozeff and Zaman (1988))

8. Venture capital share distributions (Gompers and Lerner (1995)).

There is also evidence that earnings forecasts underreact to public news, such as quarterly earnings announcements (Abarbanell and Bernard (1991, 1992), Mendenhall (1991)). An event inconsistent with this generalization is exchange listing (McConnell and Sanger (1987) and Dharan and Ikenberry (1995)). Fama (1998) argues that some of these anomalous return patterns are sensitive to empirical methodology. On the other hand, Loughran and Ritter (1998) argue that the methodology favored by Fama minimizes the power to detect possible misvaluation effects.

**Short-Term Momentum (positive short-term autocorrelation of stock returns, for individual stocks and the market as a whole):** Jegadeesh and Titman (1993), Daniel (1996); 'Short' here refers to periods on the order of 6-12 months. At very short horizons there is negative autocorrelation in individual stock returns (Jegadeesh (1990) and Lehmann (1990)), probably resulting from bid-ask spreads and other measurement problems (Kaul and Nimalendran (1990)).

Rouwenhorst (1998a) finds evidence of momentum in 12 European countries. The effect is stronger for smaller firms. However, Haugen and Baker (1996) and Daniel, Titman, and Wei (1996) show that, while there is evidence of a strong book-to-market effect in Japan, there is little or no evidence of a momentum effect. Rouwenhorst (1998a) reports a strong momentum effect within and across 12 European countries, and Rouwenhorst (1998b) finds evidence that momentum, firm size and value predict common stock returns in 20 emerging markets.

**Long-Term Reversal (negative autocorrelation of short-term returns separated by long lags, or "overreaction"):** Cross-sectionally, see DeBondt and Thaler (1985, 1987), Chopra, Lakonishok, and Ritter (1992); on robustness issues, see Fama and French (1996) and Ball, Kothari, and Shanken (1995). For the aggregate market, see Fama and French (1988) and Poterba and Summers (1988); internationally, see Richards (1997). On the robustness of the finding in the post-WWII period, see Kim, Nelson, and Startz (1988), Carmel and Young (1997), Asness (1995), and Daniel (1996); the latter two papers show that in Post-WWII US data, significant cross-sectional (Asness) and aggregate (Daniel) long-horizon negative autocorrelations are partly masked by a momentum effect (positive serial correlation) at approximately a one-year horizon.

**Unconditional Excess Volatility of Asset Prices Relative to Fundamentals:** Shiller (1981), Shiller (1989); for critical assessments of this conclusion, see Kleidon (1986) and Marsh and Merton (1986).

**Abnormal Stock Price Performance in the Opposite Direction of Long-Term Earnings Changes:** DeBondt and Thaler (1987), and Lakonishok, Shleifer, and Vishny (1994) find a negative relation between long horizon returns and past financial performance measures such as earnings or sales growth; see however DeChow and Sloan (1997). This implies that one or more short-horizon, long-lag regression coefficients must be negative (proof available on request). In contrast, Chan, Jegadeesh, and Lakonishok (1996) do not reject the null of no such a negative relation, perhaps owing to a lack of power in detecting long-run reversals. Also, La Porta, Lakonishok, Shleifer, and Vishny (1997) find large positive returns for value stocks on earnings announcement dates (and negative for growth stocks).

# Appendix B: Covariance and Variance Calculations for the Basic Model

**Covariances and Variances of Section II.B**

(All signs are under the overconfidence assumption that $\sigma_\epsilon^2 > \sigma_C^2$.)

From (6), the covariance between the date 3 and the date 2 price changes is

$$\mathrm{cov}(P_3 - P_2, P_2 - P_1) = \frac{\sigma_\theta^6 \sigma_C^2 \sigma_p^2 (\sigma_\epsilon^2 - \sigma_C^2)}{(\sigma_\theta^2 + \sigma_C^2)[\sigma_\theta^2(\sigma_C^2 + \sigma_p^2) + \sigma_C^2 \sigma_p^2]^2}. \tag{A1}$$

This is positive since $\sigma_\epsilon^2 > \sigma_C^2$.

The covariance between the date 1 price change and the date 2 price change is

$$\mathrm{cov}(P_2 - P_1, P_1 - P_0) = -\frac{\sigma_\theta^6 \sigma_C^2 (\sigma_\epsilon^2 - \sigma_C^2)}{(\sigma_\theta^2 + \sigma_C^2)^2[\sigma_\theta^2(\sigma_C^2 + \sigma_p^2) + \sigma_C^2 \sigma_p^2]}, \tag{A2}$$

which, with overconfidence, is negative. The average of the two covariances above is given by

$$\frac{\sigma_C^4 \sigma_\theta^8 (\sigma_C^2 - \sigma_\epsilon^2)}{2(\sigma_\theta^2 + \sigma_C^2)[\sigma_C^2(\sigma_\theta^2 + \sigma_p^2) + \sigma_\theta^2 \sigma_p^2]^2}, \tag{A3}$$

and is also negative. It is also easy to show that

$$\mathrm{cov}(P_3 - P_1, P_1 - \bar\theta) = -\frac{\sigma_\theta^4(\sigma_\epsilon^2 - \sigma_C^2)}{(\sigma_\theta^2 + \sigma_C^2)^2} < 0, \tag{A4}$$

$$\mathrm{cov}(P_3 - P_2, P_1 - \bar\theta) = -\frac{\sigma_\theta^4 \sigma_p^2 (\sigma_\epsilon^2 - \sigma_C^2)}{(\sigma_\theta^2 + \sigma_C^2)[\sigma_c^2(\sigma_\theta^2 + \sigma_p^2) + \sigma_\theta^2 \sigma_p^2]} < 0. \tag{A5}$$

Since $P_3 = \theta$, using the expression for $P_2$ in (6), we have

$$\mathrm{cov}(P_3 - P_2, \epsilon^*) = \frac{\sigma_\epsilon^2 \sigma_\eta^2 \sigma_\theta^2 (\sigma_\theta^2 + \sigma_p^2)(\sigma_C^2 - \sigma_\epsilon^2)}{[\sigma_\epsilon^2(\sigma_\theta^2 + \sigma_p^2) + \sigma_\theta^2 \sigma_p^2][\sigma_C^2(\sigma_\theta^2 + \sigma_p^2) + \sigma_\theta^2 \sigma_p^2]}, \tag{A6}$$

which is positive so long as $\sigma_C^2 < \sigma_\epsilon^2$.

**Proof of Proposition 3:** The variance of the date 2 price change is

$$\mathrm{var}(P_2 - P_1) = \frac{\sigma_C^4[\sigma_C^4 \sigma_\theta^6 + \sigma_\theta^8 \sigma_\epsilon^2 + \sigma_\theta^4 \sigma_p^2 (\sigma_C^2 + \sigma_\theta^2)^2]}{[\sigma_C^2(\sigma_p^2 + \sigma_\theta^2) + \sigma_\theta^2 \sigma_p^2]^2 (\sigma_\theta^2 + \sigma_C^2)^2}, \tag{A7}$$

which can either increase or decrease in $\sigma_C^2$. The date 1 price volatility,

$$\mathrm{var}(P_1 - P_0) = \frac{\sigma_\theta^4(\sigma_\theta^2 + \sigma_\epsilon^2)}{(\sigma_\theta^2 + \sigma_C^2)^2}, \tag{A8}$$

decreases with $\sigma_C^2$.

The unconditional volatility is just the average of $\mathrm{var}(P_3 - P_2)$, $\mathrm{var}(P_2 - P_1)$, and $\mathrm{var}(P_1 - P_0)$,

$$\frac{1}{3}\left[\frac{\sigma_\epsilon^2\sigma_p^4\sigma_\theta^4 + \sigma_C^4\sigma_\eta^2\sigma_\theta^2(\sigma_\eta^2+\sigma_\theta^2)}{[\sigma_C^2(\sigma_p^2+\sigma_\theta^2)+\sigma_p^2\sigma_\theta^2]^2} + \frac{\sigma_C^4\{\sigma_C^4\sigma_\theta^6 + \sigma_\theta^8\sigma_\epsilon^2 + \sigma_\theta^4\sigma_p^2(\sigma_C^2+\sigma_\theta^2)^2\}}{(\sigma_C^2+\sigma_\theta^2)^2[\sigma_C^2(\sigma_p^2+\sigma_\theta^2)+\sigma_\theta^2\sigma_p^2]^2} + \frac{\sigma_\theta^4(\sigma_\theta^2+\sigma_\epsilon^2)}{(\sigma_\theta^2+\sigma_C^2)^2}\right]. \tag{A9}$$

When there is no overconfidence, $\sigma_C^2 = \sigma_\epsilon^2$, this reduces to $\sigma_\theta^2/3$. The excess volatility is the difference between the above expression and $\sigma_\theta^2/3$, which is positive so long as there is overconfidence, $\sigma_C^2 < \sigma_\epsilon^2$.

We now calculate $\mathrm{var}(P_1 - P_0)$ and $\mathrm{var}(P_2 - P_1)$ when informed agents are rational (subscripted by $R$).

$$\mathrm{var}_R(P_1 - P_0) = \frac{\sigma_\theta^4}{\sigma_\theta^2 + \sigma_\epsilon^2} \tag{A10}$$

and

$$\mathrm{var}_R(P_2 - P_1) = \frac{\sigma_\epsilon^4\sigma_\theta^4}{(\sigma_\epsilon^2+\sigma_\theta^2)[\sigma_\epsilon^2(\sigma_p^2+\sigma_\theta^2)+\sigma_p^2\sigma_\theta^2]}. \tag{A11}$$

Using the calculated variances, $V_1^E - V_2^E$ is equal to the ratio of

$$\begin{aligned}(\sigma_\epsilon^2+\sigma_\theta^2)(\sigma_\epsilon^2-\sigma_C^2)\Big\{&\sigma_C^6(\sigma_p^2+\sigma_\theta^2)\left[\sigma_\epsilon^2(\sigma_p^2+\sigma_\theta^2)+\sigma_p^2\sigma_\theta^2\right] \\ &+\sigma_C^4\left[(\sigma_p^2+\sigma_\theta^2)(\sigma_\epsilon^4(\sigma_p^2+\sigma_\theta^2)+3\sigma_\epsilon^2\sigma_p^2\sigma_\theta^2)+2\sigma_p^4\sigma_\theta^4\right] \\ +\sigma_C^2\sigma_p^2\sigma_\theta^2&\left[2\sigma_\epsilon^4(\sigma_p^2+\sigma_\theta^2)+\sigma_\epsilon^2\sigma_\theta^2(3\sigma_p^2+2\sigma_\theta^2)+\sigma_p^2\sigma_\theta^4\right]+\sigma_\epsilon^2\sigma_p^4\sigma_\theta^4(\sigma_\epsilon^2+\sigma_\theta^2)\Big\}\end{aligned} \tag{A12}$$

and $\sigma_\epsilon^4(\sigma_C^2+\sigma_\theta^2)^2\left[\sigma_C^2(\sigma_p^2+\sigma_\theta^2)+\sigma_p^2\sigma_\theta^2)^2\right]$, and is therefore positive under overconfidence ($\sigma_\epsilon^2 > \sigma_\epsilon^2$). Thus, the proportional difference between overconfident and rational volatilities is greater at date 1 than at date 2.

**Proofs of Some Claims in Section B.3**

**Part 1 of Proposition 4:** Denote the date 2 mispricing as $M_2$. Suppressing arguments on $P_2^R(s_2)$ and $P_2^C(s_2)$, we have that $M_2 = P_2^R - P_2^C = -E[\theta - P_2^C(s_2)|s_1, s_2]$. By the properties of normal random variables, this implies that the variable $x = \theta - P_2^C + M_2$, which is the residual from the regression of $\theta - P_2^C$ on $s_1$ and $s_2$, is orthogonal to $s_1$ and $s_2$. Suppose we pick a variable $y = f(s_1, s_2)$ which is orthogonal to $M_2$. Such a variable will be orthogonal to $x$, so that we have $\mathrm{cov}(\theta - P_2^C + M_2, y) = 0$. Since $\mathrm{cov}(M_2, y) = 0$ by construction, it follows from the linearity of the covariance operator that $\mathrm{cov}(\theta - P_2^C, y) = 0$. A converse argument shows that if we pick a variable $y' = g(s_1, s_2)$ which is orthogonal to the post event return $\theta - P_2^C$ then $\mathrm{cov}(M_2, y') = 0$. Thus, all functions of $s_1$ and $s_2$ are orthogonal to $M_2$ if and only if they are orthogonal to the post event return $\theta - P_2^C$.

For the specific case when the event depends linearly on $s_2$, by (6),

$$P_3 - P_2 = \frac{\sigma_C^2\sigma_p^2\theta - \sigma_p^2\sigma_\theta^2\epsilon - \sigma_C^2\sigma_\theta^2\eta}{\sigma_\theta^2(\sigma_C^2 + \sigma_p^2) + \sigma_C^2\sigma_p^2}. \tag{A13}$$

Since $s_2 \equiv \theta+\eta$, from the above expression, it immediately follows that $\mathrm{cov}(P_3-P_2, s_2) = 0$, thus showing that events that depend only on $s_2$ are non-selective.

**Part 2 of Proposition 4:** By standard results for calculating conditional variances of normal variables (Anderson (1984)),

$$\mathrm{cov}(P_3 - P_2, s_2|s_1) = \mathrm{cov}(P_3 - P_2, s_2|P_1 - P_0) = \frac{\sigma_p^2\sigma_\theta^4(\sigma_\epsilon^2 - \sigma_C^2)}{[\sigma_C^2(\sigma_p^2 + \sigma_\theta^2) + \sigma_p^2\sigma_\theta^2](\sigma_\epsilon^2 + \sigma_\theta^2)}, \tag{A14}$$

which is positive under overconfidence ($\sigma_C^2 > \sigma_\epsilon^2$).

**Proposition 5:** Using standard normal distribution properties,

$$\epsilon^* = E[\epsilon|P_1, \theta + \eta] = \frac{\sigma_\epsilon^2(\sigma_\theta^2 + \sigma_p^2)(\theta + \epsilon) - \sigma_\theta^2\sigma_\epsilon^2(\theta + \eta)}{\sigma_\epsilon^2(\sigma_\theta^2 + \sigma_p^2) + \sigma_\theta^2\sigma_p^2}. \tag{A15}$$

It is straightforward to show that the ratio of the date 2 mispricing to $\epsilon^*$ is

$$\frac{\sigma_\epsilon^2[\sigma_C^2(\sigma_\theta^2 + \sigma_p^2) + \sigma_\theta^2\sigma_p^2]}{\sigma_p^2\sigma_\theta^2(\sigma_C^2 - \sigma_\theta^2)}, \tag{A16}$$

which is a constant (for a given level of confidence). Thus, selective events can alternatively be viewed as events that are linearly related to $\epsilon^*$.

High values of $\epsilon^*$ signify overpricing and low values underpricing. The proposition follows by observing that

$$\mathrm{cov}(P_3 - P_2, \epsilon^*) = \frac{\sigma_\epsilon^2\sigma_\eta^2\sigma_\theta^2(\sigma_\theta^2 + \sigma_p^2)(\sigma_C^2 - \sigma_\epsilon^2)}{[\sigma_\epsilon^2(\sigma_\theta^2 + \sigma_p^2) + \sigma_\theta^2\sigma_p^2][\sigma_C^2(\sigma_\theta^2 + \sigma_p^2) + \sigma_\theta^2\sigma_p^2]} < 0 \tag{A17}$$

and

$$\mathrm{cov}(P_2 - P_1, \epsilon^*) = -\frac{\sigma_C^2\sigma_\epsilon^2\sigma_\theta^4[\sigma_\epsilon^2(\sigma_p^2 + \sigma_\theta^2) + \sigma_p^2\sigma_\theta^2]}{(\sigma_C^2 + \sigma_\theta^2)[\sigma_C^2(\sigma_p^2 + \sigma_\theta^2) + \sigma_p^2\sigma_\theta^2]^2} < 0. \tag{A18}$$

Since $\mathrm{cov}(P_3 - P_2, \epsilon^*) < 0$, by the conditioning properties of mean-zero normal distributions, $E[P_3 - P_2|\epsilon^*]$ can be written in the form $k\epsilon^*$, where $k < 0$ is a constant. Thus, $E[P_3 - P_2|\epsilon^*] < 0$ if and only if $\epsilon^* > 0$. Since this holds for each positive realization of $\epsilon^*$, $E[P_3 - P_2|\epsilon^* > 0] < 0$. By symmetric reasoning, $E[P_3 - P_2|\epsilon^* < 0] > 0$. The result for event-date price reactions uses a similar method. Since $\mathrm{cov}(P_2 - P_1, \epsilon^*) < 0$, it follows that $E[P_2 - P_1|\epsilon^*] < 0$ if and only if $\epsilon^* > 0$.

**Proposition 6:** We interpret the 'fundamental/price' ratio or 'runup' as $\bar{\theta} - P_1$. For Part 1,

$$\mathrm{cov}(\bar{\theta} - P_1, \epsilon^*) = \frac{\sigma_\epsilon^2[\sigma_\epsilon^2(\sigma_p^2 + \sigma_\theta^2) + \sigma_p^2\sigma_\theta^2]}{\sigma_C^2(\sigma_p^2 + \sigma_\theta^2) + \sigma_p^2\sigma_\theta^2} > 0. \tag{A19}$$

By our assumption that the selective event is linearly related to $\epsilon^*$, the selective event is positively correlated with the mispricing measure, proving Part 1.

For Part 2, note that $\epsilon^* = k_1 s_1 + k_2 s_2$, where

$$k_1 = \frac{\sigma_\epsilon^2(\sigma_\theta^2 + \sigma_p^2)}{\sigma_C^2(\sigma_p^2 + \sigma_\theta^2) + \sigma_p^2\sigma_\theta^2}, \tag{A20}$$

$$k_2 = -\frac{\sigma_\epsilon^2\sigma_\theta^2}{\sigma_C^2(\sigma_p^2 + \sigma_\theta^2) + \sigma_p^2\sigma_\theta^2}. \tag{A21}$$

This implies that the distribution of $\epsilon^*$ conditional on $\theta + \epsilon$ is normal with mean

$$\frac{(k_1 + k_2)\sigma_\theta^2 + k_1\sigma_\epsilon^2}{\sigma_\theta^2 + \sigma_\epsilon^2}(\theta + \epsilon) \tag{A22}$$

and variance

$$\frac{[(k_1 + k_2)\sigma_\theta^2 + k_1\sigma_\epsilon^2]^2}{\sigma_\theta^2 + \sigma_\epsilon^2}(\theta + \epsilon). \tag{A23}$$

The complement of the standardized cumulative normal distribution function of a normal random variable with nonzero mean and variance is increasing in its mean. Since $E[\epsilon^*|\theta + \epsilon]$ is proportional to $\theta + \epsilon$, the probability conditional on $P_1$ that $\epsilon^*$ exceeds a given threshold value (indicating occurrence of the positive event) is increasing in $\theta + \epsilon$. The reverse holds for a negative event, proving part (2).

## Appendix C: Discrete Model of Outcome-Dependent Overconfidence

At time 0, $\theta$ has a value of $+1$ or $-1$ and an expected value of zero. At time 1, the player receives a signal $s_1$, and, at time 2, a signal $s_2$. $s_1$ may be either $H$ or $L$ while $s_2$ may be either $U$ or $D$. After each signal, the player updates his prior expected value of $\theta$.

$$Pr(s_1 = H|\theta = +1) = p = Pr(s_1 = L|\theta = -1), \tag{A24}$$

$$Pr(s_2 = U|\theta = +1) = q = Pr(s_2 = D|\theta = -1). \tag{A25}$$

The probabilities that $\theta = +1$, given $s_1$ and $s_2$ are

$$\begin{aligned} Pr(\theta = +1|s_1 = H) &= \frac{Pr(s_1 = H|\theta = +1)Pr(\theta = +1)}{Pr(s_1 = H)} \\ &= \frac{p/2}{p/2 + (1-p)/2} = p. \end{aligned} \tag{A26}$$

When $s_2$ confirms $s_1$ (either $s_1 = H, s_2 = U$ or $s_1 = L, s_2 = D$), the player becomes overconfident and acts as if his precision were $p_C$ instead of $p$, so

$$
\begin{aligned}
Pr(\theta = +1 | s_1 = H, s_2 = U) &= \frac{Pr(s_1 = H, s_2 = U | \theta = +1) Pr(\theta = +1)}{Pr(s_1 = H, s_2 = U)} \\
&= \frac{p_C q}{p_C(2q-1) + (1-q)}. \qquad \text{(A27)}
\end{aligned}
$$

When $s_2$ is informative ($q > 1/2$), this probability exceeds $p_C$. When $s_2$ does not confirm $s_1$, the player does not become overconfident, so

$$
\begin{aligned}
Pr(\theta = +1 | s_1 = H, s_2 = D) &= \frac{Pr(s_1 = H, s_2 = D | \theta = +1) Pr(\theta = +1)}{Pr(s_1 = H, s_2 = D)} \\
&= \frac{p(1-q)}{p(1-q) + q(1-p)}. \qquad \text{(A28)}
\end{aligned}
$$

When evaluated with an informative signal $s_2$ ($q > 1/2$), this probability is less than $p$. With a risk neutral player, the price of the asset with value $\theta$ can be calculated linearly using the above probabilities. The price at time 0 ($P_0$) is, by definition, equal to 0. As $\theta$ can take on a value of $+1$ or $-1$, the price is $(\rho)(+1) + (1-\rho)(-1)$ or, $2\rho - 1$, where $\rho$ is the probability that $\theta$ is $+1$.

$$
\begin{aligned}
P_1|_{s_1=H} = -P_1|_{s_1=L} &= 2Pr(\theta = +1 | s_1 = H) - 1 = 2p - 1 \qquad \text{(A29)} \\
P_2|_{s_1=H,s_2=U} = -P_2|_{s_1=L,s_2=D} &= 2Pr(\theta = +1 | s_1 = H, s_2 = U) - 1 \\
&= \frac{p_C + q - 1}{p_C(2q-1) + (1-q)} \qquad \text{(A30)} \\
P_2|_{s_1=H,s_2=D} = -P_2|_{s_1=L,s_2=U} &= 2Pr(\theta = +1 | s_1 = H, s_2 = D) - 1 \\
&= \frac{p - q}{p + q - 2pq}. \qquad \text{(A31)}
\end{aligned}
$$

The price changes are $\Delta P_1 = P_1 - P_0 = P_1$ and $\Delta P_2 = P_2 - P_1$. $E[P_1] = 0$, so $\text{cov}(\Delta P_1, \Delta P_2) = E[\Delta P_1 \Delta P_2]$. The probabilities of the eight possible outcomes are:

$$
\begin{aligned}
Pr(\theta = +1, s_1 = H, s_2 = U) = Pr(\theta = -1, s_1 = L, s_2 = D) &= pq/2 \qquad \text{(A32)} \\
Pr(\theta = -1, s_1 = H, s_2 = U) = Pr(\theta = +1, s_1 = L, s_2 = D) &= (1-p)(1-q)/2 \qquad \text{(A33)} \\
Pr(\theta = +1, s_1 = H, s_2 = D) = Pr(\theta = -1, s_1 = L, s_2 = U) &= p(1-q)/2 \qquad \text{(A34)} \\
Pr(\theta = -1, s_1 = H, s_2 = D) = Pr(\theta = +1, s_1 = L, s_2 = U) &= (1-p)q/2. \qquad \text{(A35)}
\end{aligned}
$$

The product $\Delta P_1 \Delta P_2$ can only take on two values, based upon the various signal combinations:

$$
X \equiv [\Delta P_1 \Delta P_2]_{s_1=H,s_2=U} = [\Delta P_1 \Delta P_2]_{s_1=L,s_2=D}
$$

$$= (2p-1)\left(\frac{p_C+q-1}{p_C(2q-1)+(1-q)}-(2p-1)\right). \quad \text{(A36)}$$

$$Y \equiv [\Delta P_1 \Delta P_2]_{s_1=H,s_2=D} = [\Delta P_1 \Delta P_2]_{s_1=L,s_2=U}$$

$$= (2p-1)\left(\frac{p-q}{p+q-2p}-(2p-1)\right). \quad \text{(A37)}$$

Combining, $E[\Delta P_1 \Delta P_2]$ can be written as $(1-a)X+aY$ where $a=p+q-2pq$. After some calculation, the two components of this expression become:

$$(1-a)X = \frac{2(2p-1)(2pq-p-q+1)(p_C p+p_C q+pq-2p_C pq-p)}{p_C(2q-1)+(1-q)} \quad \text{(A38)}$$

$$aY = 2p(2q-1)(2p-1)(p-1). \quad \text{(A39)}$$

Combining these two terms and a great deal of factoring produces the final result,

$$E[\Delta P_1 \Delta P_2] = \frac{2q(2p-1)(p_C-p)(1-q)}{p_C(2q-1)+(1-q)} > 0. \quad \text{(A40)}$$

When there is no overconfidence ($p_C = p$) this expression is zero and price changes are uncorrelated.

**A Second Noisy Public Signal**

The model so far shows that overreaction can be exaggerated by a possible rise in confidence triggered by a noisy public signal. We now add a second noisy public signal to consider whether correction of mispricing is gradual. Signal $s_{3'}$ follows $s_2$ and can take on values $G$ or $B$. The precision of this signal is as follows:

$$Pr(s_{3'}=G|\theta=+1)=r=Pr(s_{3'}=B|\theta=-1). \quad \text{(A41)}$$

This signal does not affect confidence. If the player becomes overconfident (and replaces $p$ with $p_C$) after $s_2$, then the player will continue to use $p_C$ as his measure of the precision of $s_1$, regardless of whether $s_{3'}$ confirms $s_1$. As there are two possible prices after the first signal and four possible prices after the second, there are eight possible prices after observation of the third signal. As above, by symmetry, only half of these prices need to be calculated. Using the conditional probabilities, the period three prices are:

$$P_{3'}|_{s_1=H,s_2=U,s_{3'}=G} = \frac{p_C qr-(1-p_C)(1-q)(1-r)}{p_C qr+(1-p_C)(1-q)(1-r)}; \quad \text{(A42)}$$

$$P_{3'}|_{s_1=H,s_2=U,s_{3'}=B} = \frac{p_C q(1-r)-(1-p_C)(1-q)r}{p_C q(1-r)+(1-p_C)(1-q)r}; \quad \text{(A43)}$$

$$P_{3'}|_{s_1=H,s_2=D,s_{3'}=G} = \frac{p(1-q)r-(1-p)q(1-r)}{p(1-q)r+(1-p)q(1-r)}; \quad \text{(A44)}$$

$$P_{3'}|_{s_1=H,s_2=D,s_{3'}=B} = \frac{p(1-q)(1-r)-(1-p)qr}{p(1-q)(1-r)+(1-p)qr}. \quad \text{(A45)}$$

With two possible values for $\theta$, there are now sixteen possible sets of $\{\theta, s_1, s_2, s_{3'}\}$ realizations. Only $\{s_1, s_2, s_{3'}\}$ are observed by the player, resulting in eight sets of possible signal realizations. When calculating the covariances of price changes, only half of these realizations can result in unique products of price changes, so we define

$$A_{ij} \equiv \Delta P_i \Delta P_j|_{H,U,G} = \Delta P_i \Delta P_j|_{L,D,B}; \tag{A46}$$

$$B_{ij} \equiv \Delta P_i \Delta P_j|_{H,U,B} = \Delta P_i \Delta P_j|_{L,D,G}; \tag{A47}$$

$$C_{ij} \equiv \Delta P_i \Delta P_j|_{H,D,G} = \Delta P_i \Delta P_j|_{L,U,B}; \tag{A48}$$

$$D_{ij} \equiv \Delta P_i \Delta P_j|_{H,D,B} = \Delta P_i \Delta P_j|_{L,U,G}. \tag{A49}$$

Each of these four possible products must then be weighted by their probability of occurrence to calculate the expected value of the products of the price changes (the expected value of each price is zero). The weights for the $A_{ij}$ component of covariance are:

$$Pr(H,U,G|\theta=+1) + Pr(H,U,G|\theta=-1) = pqr/2 + (1-p)(1-q)(1-r)/2, \tag{A50}$$

$$Pr(L,D,B|\theta=-1) + Pr(L,D,B|\theta=-1) = pqr/2 + (1-p)(1-q)(1-r)/2. \tag{A51}$$

Proceeding in this manner, the covariances are:

$$\begin{aligned} E[\Delta P_i \Delta P_j] = {} & [pqr + (1-p)(1-q)(1-r)]A_{ij} + [pq(1-r) + (1-p)(1-q)r]B_{ij} + \\ & [p(1-q)r + (1-p)q(1-r)]C_{ij} + [p(1-q)(1-r) + (1-p)qr]D_{ij}. \end{aligned} \tag{A52}$$

(Earlier calculations of $E[\Delta P_1 \Delta P_2]$ had $A_{12} = B_{12} = X$ and $C_{12} = D_{12} = Y$, with the $r$ and $1-r$ factors from $s_{3'}$ summing to one.) To simplify the algebra, temporarily let all signals have the same precision (i.e., $q = r = p$), with $p_C$ replacing $p$ as the perceived precision of the first signal if overconfidence occurs. Direct calculation of the covariances then shows that

$$E[\Delta P_1 \Delta P_2]_{r=q=p} = \frac{2p(1-p)(2p-1)(p_C-p)}{p_C(2p-1)+1-p} > 0; \tag{A53}$$

$$E[\Delta P_1 \Delta P_{3'}]_{r=q=p} = \frac{2pp_C(p-1)(p_C-p)(1-p_C)(2p-1)^3}{[p_C(2p-1)+1-p][p_C(2p-1)+(1-p)^2]} < 0; \tag{A54}$$

$$E[\Delta P_2 \Delta P_{3'}]_{r=q=p} = \frac{4p^2 p_C(p-1)^2(p_C-p)(p_C-1)(2p-1)^2(2p_C-1)}{[p_C(2p-1)+1-p]^2[p_C(2p-1)+(1-p)^2]} < 0. \tag{A55}$$

By direct comparison, $E[\Delta P_1 \Delta P_2]_{r=q=p}$ and $E[\Delta P_2 \Delta P_{3'}]_{r=q=p}$ are related by:

$$E[\Delta P_2 \Delta P_{3'}]_{r=q=p} = -\frac{2p(1-p)p_C(1-p_C)(2p-1)(2p_C-1)}{[p_C(2p-1)+(1-p)][p_C(2p-1)+(1-p)^2]} E[\Delta P_1 \Delta P_2]_{r=q=p}, \tag{A56}$$

so the covariance between the date 2 and 3 price changes is negatively proportional to the covariance of the date 1 and 2 price changes. Consider the numerator $N$ of the proportionality factor. The first three components, $2p(1-p)$, are maximized when $p = 1/2$ while the next two components, $p_C(1-p_C)$, are maximized when $p_C = 1/2$. Since the last two components satisfy $(2p-1)(2p_C-1) < 1$, the $N \leq 1/8$. In the denominator $D$, the expression $p_C(2p-1) + (1-p)$ is minimized when $p_C = p = 1/2$, resulting in a minimum of $1/2$. The second component of $D$ is similarly minimized when $p_C = p = 1/2$, resulting in a minimum of $1/4$. So $D \geq 1/8$. Since $N \leq 1/8$, the ratio $N/D \leq 1$. Therefore, the negative covariance between date two and date three price changes must be, in absolute value, less than or equal to the positive covariance between period one and period two price changes, resulting in an overall one-period covariance that is positive.

When $q = r$ differs from $p$, direct calculation of covariances shows:

$$E[\Delta P_1 \Delta P_2]_{r=q} > 0; \tag{A57}$$

$$E[\Delta P_1 \Delta P_{3'}]_{r=q} < 0; \tag{A58}$$

$$E[\Delta P_2 \Delta P_{3'}]_{r=q} < 0. \tag{A59}$$

Now let the signal $s_{3'}$ have a precision of $r$ that differs from both precisions of $p$ and $q$. Proceeding as above, the covariances satisfy

$$E[\Delta P_1 \Delta P_2] > 0; \tag{A60}$$

$$E[\Delta P_1 \Delta P_{3'}] < 0; \tag{A61}$$

$$E[\Delta P_2 \Delta P_{3'}] < 0; \tag{A62}$$

$$E[\Delta P_{3'} \Delta P_3] > 0; \tag{A63}$$

$$E[\Delta P_1 \Delta P_3] < 0, \tag{A64}$$

The magnitude of $E[\Delta P_2 \Delta P_{3'}]$ varies non-monotonically with $q$. As $r$ rises (the precision of $s_{3'}$ is increased), direct calculation shows that $E[\Delta P_2 \Delta P_{3'}]$ becomes more negative (increases in absolute value). As $r \to 0.5$, this covariance approaches zero. Thus, when the second noisy public signal is not very informative, this negative single-lag covariance becomes arbitrarily small in absolute value.

Confidence increases when $s_2$ confirms $s_1$, but its effects are mitigated as $s_2$ becomes more informative. Thus, an increase in the precision of $s_2$ has an ambiguous effect on $E[\Delta P_2 \Delta P_{3'}]$. This increase results in a greater likelihood of overconfidence occurring, yet also places greater, rational, confidence in $s_2$ itself, yielding less leverage to the effects of overconfidence. (At the extreme, a value of $q$ equal to one yields the greatest chances of $s_2$ confirming $s_1$ yet results in zero values for all covariances as the perfect information of $s_2$

entirely determines all subsequent prices.) Based on simulation, it appears that the greater information resulting from higher values of $q$ tends to over-shadow the increased likelihood of overconfidence, resulting in generally lower absolute values for $E[\Delta P_2 \Delta P_{3'}]$.

Larger values of $r$, the precision of $s_{3'}$, result in more negative values of $E[\Delta P_2 \Delta P_{3'}]$. In this case, a more informative second noisy public signal can only place less weight on previous signals and result in a stronger correction of the previous overreaction. Thus, the final one-period covariance is more negative as the precision of $s_{3'}$ rises.

## Appendix D: Covariance Calculations for the Dynamic Model (Section III, Subsection A.1)

Since the probability $p$ is an exogenous constant, and the probability that the date 1 price move was positive is 1/2, by the law of iterated expectations

$$\begin{aligned} \mathrm{cov}(P_2 - P_1, P_1 - P_0) &= E_{s_2}[E\left[\{(P_2 - P_1)(P_1 - P_0)\} \,|s_2\right]] \\ &= \frac{k\sigma_\theta^4(\sigma_\theta^2 + \sigma_\epsilon^2)}{2(\sigma_\theta^2 + \sigma_C^2)^2(\sigma_\theta^2 + \sigma_C^2 - k)} > 0. \qquad \text{(A65)} \\ \mathrm{cov}(P_3 - P_2, P_2 - P_1) &= E_{s_2}\left[E\left\{\{(P_3 - P_2)(P_2 - P_1)\} \,|s_2\right\}\right] \\ &= -\frac{\sigma_\theta^2[k^2\sigma_\theta^2 + k(\sigma_\epsilon^2 - \sigma_C^2)]}{2(\sigma_\theta^2 + \sigma_C^2)(\sigma_\theta^2 + \sigma_C^2 - k)^2} < 0. \qquad \text{(A66)} \end{aligned}$$

Further,

$$\mathrm{cov}(P_3 - P_1, P_1 - P_0) = E[(\theta - P_1)P_1] = -\frac{\sigma_\theta^4(\sigma_\epsilon^2 - \sigma_C^2)}{(\sigma_\theta^2 + \sigma_C^2)^2} < 0. \qquad \text{(A67)}$$

Direct calculation shows that

$$\begin{aligned} \mathrm{cov}(P_3 - P_{3'}, P_{3'} - P_2) = \frac{\sigma_\theta^6}{2}\Bigg[&\frac{\sigma_C^2(\sigma_\epsilon^2 - \sigma_C^2)}{(\sigma_\theta^2 + \sigma_C^2)^2[\sigma_\theta^2(\sigma_C^2 + \sigma_p^2) + \sigma_C^2\sigma_p^2]} \\ &+ \frac{(\sigma_C^2 - k)(\sigma_\epsilon^2 + k - \sigma_C^2)}{(\sigma_\theta^2 + \sigma_C^2 - k)^2[\sigma_\theta^2(\sigma_C^2 - k + \sigma_p^2) + (\sigma_C^2 - k)\sigma_p^2]}\Bigg] > 0; \qquad \text{(A68)} \end{aligned}$$

$$\mathrm{cov}(P_3 - P_3', P_2 - P_1) = -\frac{k\sigma_p^2\sigma_\theta^4(k + \sigma_\epsilon^2 - \sigma_C^2)}{2(\sigma_C^2 - k + \sigma_\theta^2)[(\sigma_C^2 - k)(\sigma_\theta^2 + \sigma_p^2) + \sigma_p^2\sigma_\theta^2]} < 0; \qquad \text{(A69)}$$

$$\begin{aligned} \mathrm{cov}(P_3 - P_3', P_1 - P_0) = -\frac{1}{2}\Bigg[&\frac{\sigma_\theta^4\sigma_p^2(\sigma_\epsilon^2 - \sigma_C^2)}{(\sigma_\theta^2 + \sigma_C^2)[\sigma_C^2(\sigma_\theta^2 + \sigma_p^2) + \sigma_\theta^2\sigma_p^2]} \\ &+ \frac{\sigma_\theta^4\sigma_p^2(k + \sigma_\epsilon^2 - \sigma_C^2)}{(\sigma_\theta^2 + \sigma_C^2)[(\sigma_C^2 - k)(\sigma_\theta^2 + \sigma_p^2) + \sigma_p^2\sigma_\theta^2]}\Bigg] < 0; \qquad \text{(A70)} \end{aligned}$$

$$\mathrm{cov}(P_3' - P_2, P_1 - P_0) = -\frac{1}{2}\Bigg[\frac{\sigma_\theta^6\sigma_C^2(\sigma_\epsilon^2 - \sigma_C^2)}{(\sigma_\theta^2 + \sigma_p^2)^2[\sigma_C^2(\sigma_\theta^2 + \sigma_p^2) + \sigma_\theta^2\sigma_p^2]}$$

$$+\frac{\sigma_\theta^6(\sigma_C^2-k)(k+\sigma_\epsilon^2-\sigma_C^2)}{(\sigma_\theta^2+\sigma_C^2-k)[(\sigma_C^2-k)(\sigma_\theta^2+\sigma_p^2)+\sigma_p^2\sigma_\theta^2]}\Bigg] < 0; \tag{A71}$$

and that

$$\mathrm{cov}(P_{3'}-P_2, P_2-P_1) = -\frac{k\sigma_\theta^6(\sigma_C^2-k)(\sigma_\epsilon^2+k-\sigma_C^2)}{2(\sigma_C^2+\sigma_\theta^2-k)^2[\sigma_p^2\sigma_\theta^2+(\sigma_p^2+\sigma_\theta^2)(\sigma_C^2-k)]} < 0. \tag{A72}$$

The three single-lag contiguous covariances are given by (A65), (A68), and (A72). Comparing the covariances (A65) and (A72), it is evident that the sum of these two covariances will be negative so long as $\sigma_p^2$ is sufficiently large since the latter (negative) covariance varies inversely with $\sigma_p^2$ and the former covariance does not depend on $\sigma_p^2$. Since the covariance in (A68) is always positive, unconditional momentum (defined as the simple arithmetic average of the three single covariances) obtains if $\sigma_p^2$ is sufficiently small.

As $k \to 0$, the covariance in (A72) goes to zero, while the covariance in (A68) remains strictly positive. Thus, if $k$ is sufficiently small, the sum of these two covariances will be positive. Since (A65) is always nonnegative, unconditional momentum will obtain if $k$ is sufficiently small.

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# Footnotes

[1]A recent revision of Odean's paper offers a modified model that allows for underreaction. This is developed in a static setting with no public signals, and therefore does not address issues such as short-term versus long-term return autocorrelations, and event study anomalies.

[2]See respectively: Oskamp (1965); Christensen-Szalanski and Bushyhead (1981), Baumann, Deber, and Thompson (1991); Kidd (1970); Wagenaar and Keren (1986); Neale and Bazerman (1990); Cooper, Woo, and Dunkelberg (1988); Russo and Schoemaker (1992); Stael von Holstein (1972); Ahlers and Lakonishok (1983), Elton, Gruber, and Gultekin (1984), Froot and Frankel (1989), DeBondt and Thaler (1990), DeBondt (1991). See Odean (1998) for a good summary of empirical research on overconfidence.

[3]Odean (1998) (Section II.D) also makes a good argument for why overconfidence should dominate in financial markets. Also, Bernardo and Welch (1998) offer an evolutionary explanation for why individuals should be overconfident

[4]Greenwald (1980), Svenson (1981), Cooper, Woo, and Dunkelberg (1988), and Taylor and Brown (1988).

[5]See Alpert and Raiffa (1982); Fischhoff, Slovic, and Lichtenstein (1977); Batchelor and Dua (1992); and the discussions of Lichtenstein, Fischoff, and Phillips (1982) and Yates (1990).

[6]Some previous models with common private signals include Grossman and Stiglitz (1980), Admati and Pfleiderer (1988), and Hirshleifer, Subrahmanyam, and Titman (1994). If some analysts and investors use the same information sources to assess security values, and interpret them in similar ways, the error terms in their signals will be correlated. For simplicity, we assume this correlation is unity; however, similar results would obtain under imperfect (but nonzero) correlation in signal noise terms.

[7]It is not crucial for the analysis that the $U$s correctly assess the private signal variance, only that they do not underestimate it as much as the informed do. Also, since the uninformed do not possess a signal to be overconfident about, they could alternatively be

interpreted as fully rational traders who trade to exploit market mispricing. Furthermore, most of the results will obtain even if investors are symmetrical both in their overconfidence and their signals. Results similar to those we derive would apply in a setting where identical overconfident individuals receive correlated private signals.

[8]We thank an anonymous referee for suggesting we explore this issue.

[9]The initial positive return relative to issue price, 'underpricing,' is *not* an announcement reaction to the news that an IPO will occur; this news is released earlier.

[10]The model's event study predictions also apply to events undertaken by outsiders who have information about the firm. An example is an analyst's recommendation to buy or sell shares of the firm. Thus, the analysis is consistent with evidence on stock price drift following analyst 'buy' and 'sell' recommendations mentioned in Appendix A.

[11]Either concave utility or risk of dismissal can make a manager averse to a low stock price; a rising disutility from low price is a common model assumption (see, *e.g.*, Harris and Raviv (1985)). If managers prefer a high short-term stock price but risk incurring a penalty for over-aggressive reports, then the net benefit from reporting higher earnings may be greater, *ceteris paribus*, when the stock is more undervalued.

[12]Proposition 1 is based on a non-selective news event, namely, the arrival of $s_2$. Even though $s_2$ is private information here, the result is the same because $s_2$ is fully revealed by the corporate action, so that $P_2$ is identical in all states to what it would be if $s_2$ were made public directly. Thus, $\mathrm{cov}(P_3 - P_2, P_2 - P_1)$ is the same in both cases.

[13]Fama (1998) argues that our approach implies that mean pre-event abnormal returns will have the same sign as mean post-event abnormal returns, and that the evidence does not support this implication. As discussed above, event occurrence is likely to depend on past public information, in which case the model implies that average pre-event runup can have either the same or the opposite sign as average post-event abnormal returns. See Propositions 4 and 5 for model implications for event study returns that are robust with respect to pre-event public information arrival. The evidence generally supports these predictions.

[14]However, Jung, Kim, and Stulz (1996) find that firms often depart from the pecking

order ($i.e.$, the preference of debt over equity) because of agency considerations, and that debt and equity issuers both have negative average abnormal long-run stock returns which are not statistically different from one another.

[15]Formally, $\mathrm{cov}(P_2 - P_1, P_1 - P_0) > 0$, $\mathrm{cov}(P_3 - P_{3'}, P_{3'} - P_2) > 0$, and $\mathrm{cov}(P_{3'} - P_2, P_2 - P_1) < 0$.

[16]Several alternative *ad hoc* updating rules consistent with this intuition all lead to roughly equivalent results.

[17]For tractability, we assume that the investor forms beliefs as if, at each point in time, he knows his exact signal precision. Rationally he should allow for the fact that $v_{C,t}$ is an estimate. We expect that the essential results are not sensitive to this simplification.

[18]The discussion of event-study implications in Subsection B.3 described conditions under which post-earnings announcement drift could be an underreaction effect.

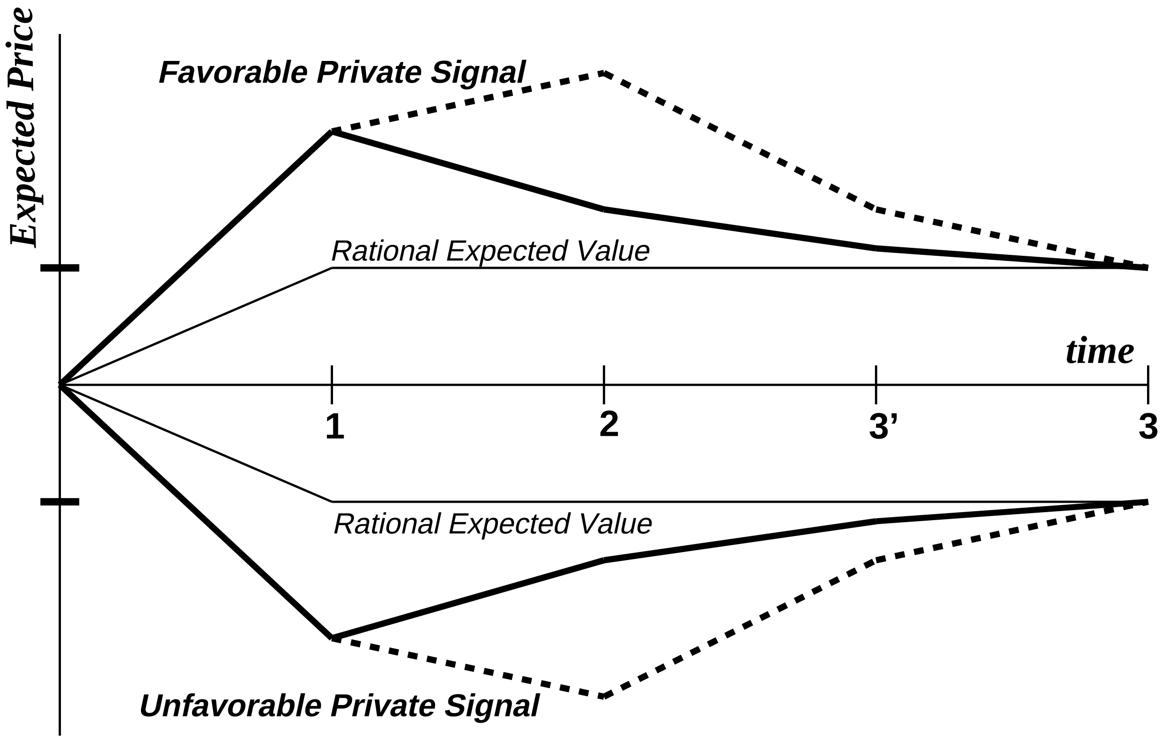


Figure 1: **Average Price as a Function of Time with Overconfident Investors**

This Figure shows price as a function of time for the dynamic model of Section III with (dashed line) and without (solid line) self-attribution bias.

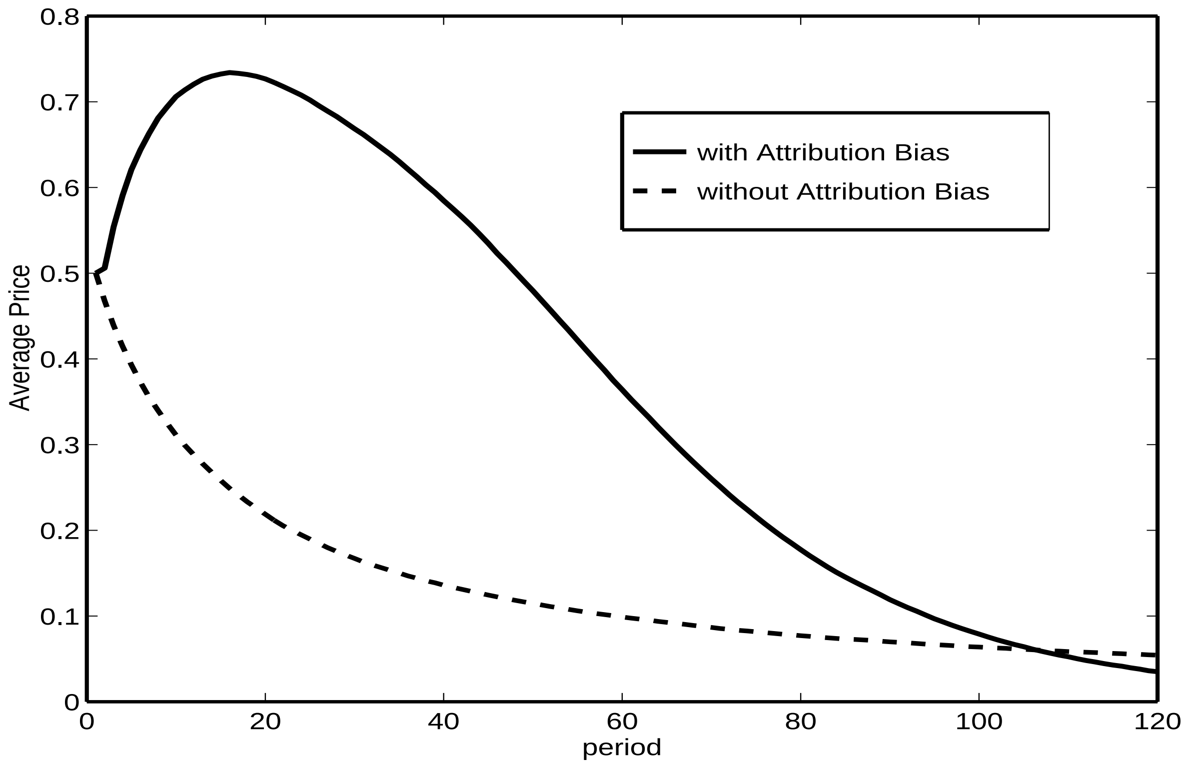


Figure 2: **Average Price Path Following Private Information Shock**

This Figure shows average price path calculated using the simulation in Section III.B.3, following a private information shock $s_1 = 1$. The price-path is shown for the dynamic model with (solid line) and without (dashed line) self-attribution bias.

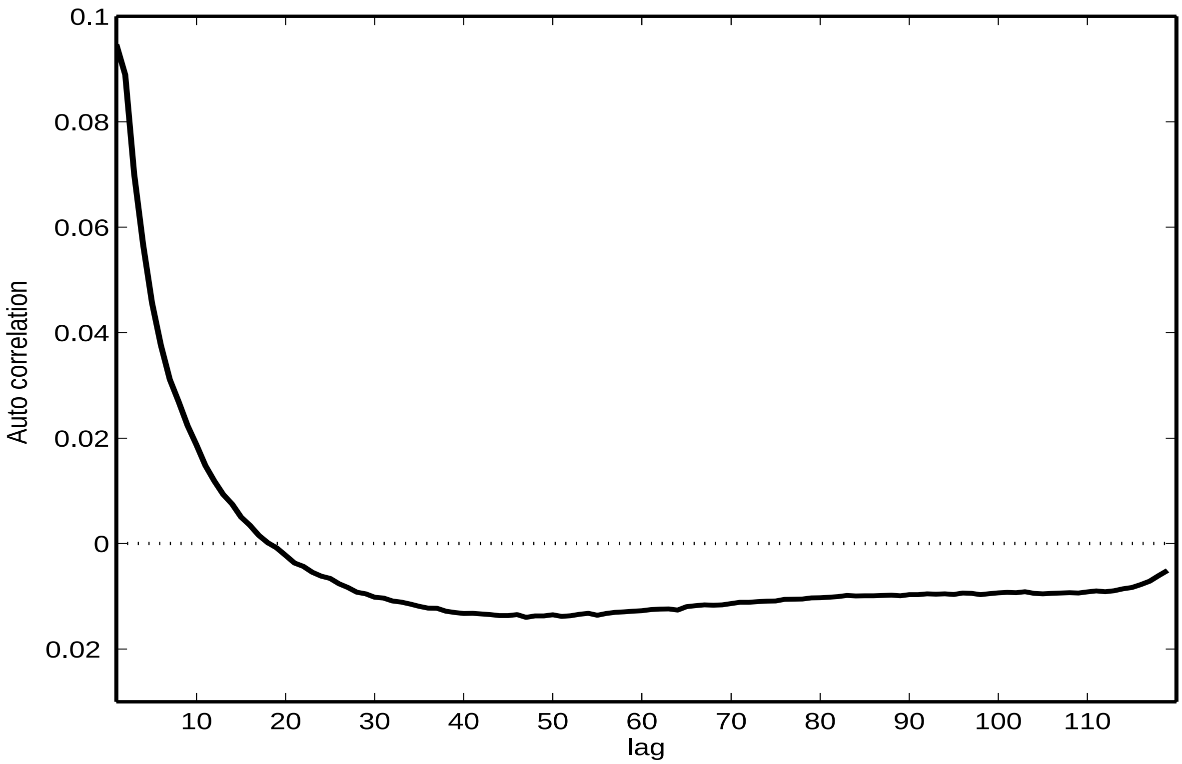


Figure 3: **Average Price-Change Autocorrelations**

This Figure presents the unconditional average autocorrelations (at lags between 1 period and 119 periods), calculated using the simulation described in Section III.B.3.

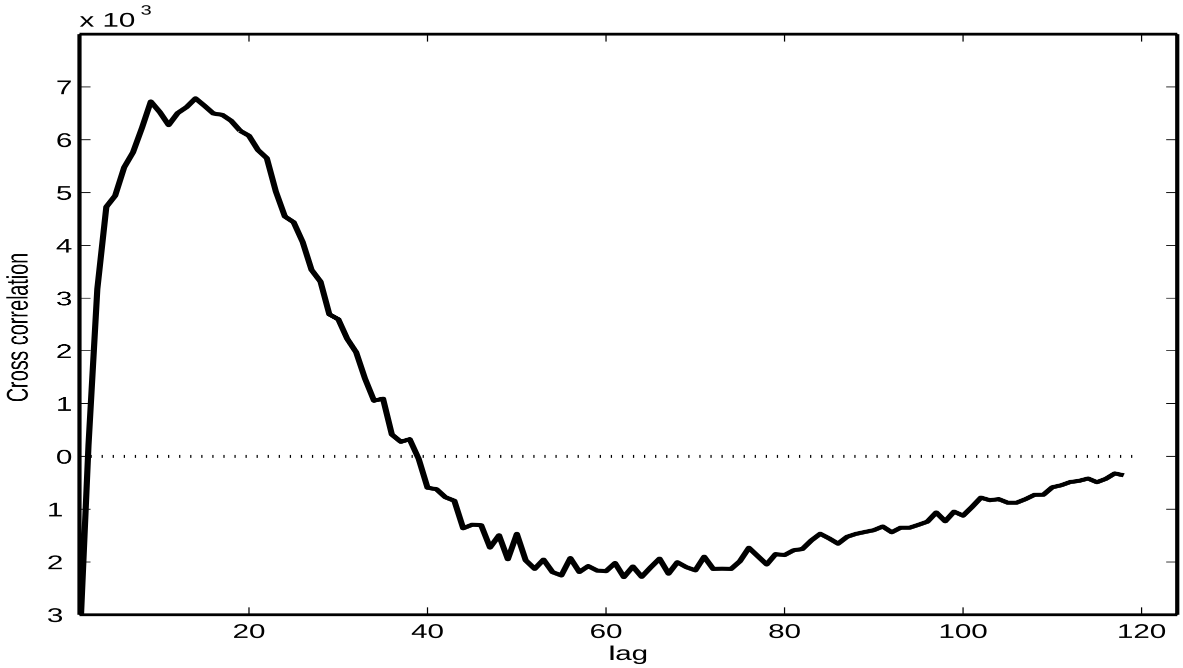


Figure 4: **Correlation between Information Changes and Future Price Changes**

This Figure shows the set of average sample correlations between the $\Delta e_t$ and price changes $\tau$ periods in the future $\Delta P_{t+\tau} = P_{t+\tau} - P_{t+\tau-1}$. These are calculated using the simulated dynamic model of Section III.B.3.