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Momentum Crashes

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- Abstract -

Across numerous asset classes, momentum strategies have historically generated high Sharpe ratios and strong positive alphas relative to standard asset pricing models. However, the returns to momentum strategies are negatively skewed: they experience infrequent but strong and persistent strings of negative returns. These momentum crashes are partly forecastable. They occur in what we term "panic" states – following market declines and when market volatility is high, and are contemporaneous with market "rebounds." We show that the low *ex-ante* expected returns in panic states result from a conditionally high premium attached to the option-like payoffs of past losers. An implementable dynamic momentum strategy based on forecasts of each momentum strategy's mean and variance generates an unconditional Sharpe ratio approximately double that of the static momentum strategy. Further, we show that momentum returns in panic states are correlated with, but not explained by, volatility risk. These results are robust across eight different markets and asset classes and multiple time periods.

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1 Introduction

A momentum strategy is a bet that past returns will predict future returns in the cross-section of assets, and is typically implemented by buying past winners and selling past losers. Momentum is pervasive: the academic finance literature documents the efficacy of momentum strategies across multiple time periods, across many markets, and in numerous asset classes that include equities, bonds, currencies, commodities, and exchange-traded futures.[1] Momentum is a strategy employed by numerous quantitative investors within and across multiple asset classes and even by mutual fund managers.[2]

Despite the pervasive evidence of momentum, the underlying mechanism responsible for its returns is as yet unknown. By virtue of the high Sharpe ratios associated with momentum strategies, the return patterns are difficult to explain within the standard rational-expectations asset pricing framework. Following Hansen and Jagannathan (1991), in a frictionless framework the high Sharpe ratio associated with zero-investment momentum portfolios implies high variability of marginal utility across states of nature. Moreover, the lack of correlation of momentum portfolio returns with standard proxy variables for macroeconomic risk (*e.g.,* consumption growth) sharpens the puzzle still further (see, *e.g.*, Daniel and Titman (2012)).

[1] Momentum strategies were first documented in U.S. common stock returns from 1965 to 1989 by Jegadeesh and Titman (1993) and Asness (1995), by sorting firms on the basis of three to 12 month past returns. Subsequently, Jegadeesh and Titman (2001) show the continuing efficacy of US equity momentum portfolios in common stock returns in the 1990 to 1998 period. Israel and Moskowitz (2013) show the robustness of momentum prior to and after these studies from 1927 to 1965 and from 1990 to 2012. There is even evidence of momentum going back to the Victorian age from Chabot, Remy, and Jagannathan (2009) and evidence from 1801 to 2012 from Geczy and Samonov (2013) in what the authors call "the world's longest backtest." Strong and persistent momentum effects are also present outside of the US equity market. Rouwenhorst (1998) finds evidence of momentum in equities in developed markets, and Rouwenhorst (1999) documents momentum in emerging markets. Asness, Liew, and Stevens (1997) demonstrate positive momentum in country indices. Among common stocks, there is also evidence that momentum strategies perform well for industry strategies, and for strategies that are based on the firm specific component of returns (Moskowitz and Grinblatt (1999), Grundy and Martin (2001), and Asness, Porter, and Stevens (2000)). Momentum is also present outside of equities: Okunev and White (2003) find momentum in currencies; Erb and Harvey (2006) in commodities; Moskowitz, Ooi, and Pedersen (2012) in exchange traded futures contracts; and Asness, Moskowitz, and Pedersen (2013) in bonds. Asness, Moskowitz, and Pedersen (2013) also integrate the evidence on within-country cross-sectional equity, country-equity, country-bond, currency, and commodity momentum strategies.

[2] Jegadeesh and Titman (1993) motivate their investigation of momentum with the observation that "...a majority of the mutual funds examined by Grinblatt and Titman (1989, 1993) show a tendency to buy stocks that have increased in price over the previous quarter." Grinblatt, Titman, and Wermers (1995) further document the prevalence of momentum-based strategies in mutual fund holdings. Asness, Ilmanen, Israel, and Moskowitz (2013) show that hedge funds exhibit significant exposure to momentum across a variety of hedge fund categories.

A number of behavioral theories of price formation purport to yield momentum as an implication, such as Daniel, Hirshleifer and Subrahmanyam (1998, 2001), Barberis, Shleifer, and Vishny (1998), Hong and Stein (1999), Grinblatt and Han (2005), and Frazzini (2006), but no single theory has been accepted as a definitive explanation.

However, the strong positive average returns and Sharpe ratios of momentum strategies are punctuated with occasional strong reversals, or "crashes." Like the returns to the carry trade in currencies, momentum returns are negatively skewed, and the crashes can be pronounced and persistent.[3] In our 1927 to 2013 U.S. equity sample, the two worst months for a momentum strategy that buys the top decile of past 12-month winners and shorts the bottom decile of losers are consecutive: July and August of 1932. Over this short period, the past-loser decile portfolio returned 232%, while the past-winner decile portfolio had a gain of only 32%. In a more recent crash, over the three-month period from March to May of 2009, the past-loser decile rose by 163%, while the decile portfolio of past winners gained only 8%.

We investigate the impact of and potential predictability of these momentum crashes, which appear to be a key and robust feature of momentum strategies. At the start of each of the two crashes discussed above (July/August of 1932 and March-May of 2009), the broad US equity market was down significantly from earlier highs. Market volatility was also high. And, importantly, the market as a whole rebounded significantly during these momentum crash months. This is consistent with what we find regarding the general behavior of momentum crashes: they tend to occur in times of market stress, specifically when the market has fallen and when *ex-ante* measures of volatility are high. They also occur when contemporaneous market returns are high.[4]

These patterns are suggestive of the possibility that the changing beta of the momentum portfolio may partly be driving the momentum crashes. The time variation in betas of return sorted portfolios was first documented by Kothari and Shanken (1992), who argue that, by their nature, past-return sorted portfolios will have significant time-varying exposure to systematic factors. Because momentum strategies are bets on past winners, they will have positive loadings on factors which have had a positive realization over the formation period

[3]See Brunnermeier, Nagel, and Pedersen (2008), and others for evidence on the negative skewness of carry trade returns.

[4]Our result is consistent with that of Cooper, Gutierrez, and Hameed (2004) and Stivers and Sun (2010), who find, respectively, that the momentum premium falls to zero when the past three-year market return has been negative and that the momentum premium is low when market volatility is high. However, these papers do not examine conditional risk measures as we do here.

of the momentum strategy.

Grundy and Martin (2001) apply Kothari and Shanken's insights to price momentum strategies. Intuitively, the result is straightforward, if not often appreciated: when the market has fallen significantly over the momentum formation period – in our case from 12 months ago to 1 month ago – there is a good chance that the firms that fell in tandem with the market were and are high beta firms, and those that performed the best were low beta firms. Thus, following market declines, the momentum portfolio is likely to be long low-beta stocks (the past winners), and short high-beta stocks (the past losers). We verify empirically that there is dramatic time variation in the betas of momentum portfolios. Using beta estimates based on daily returns we find that, following major market declines, betas for the past-loser decile rise above 3, and fall below 0.5 for past winners. Hence, when the market rebounds quickly, momentum strategies will crash because they have a conditionally large negative beta.

Grundy and Martin (2001) further argue that performance of the momentum portfolio is dramatically improved, particularly in the pre-WWII era, by dynamically hedging the market and size risk in the portfolio. However, their hedged portfolio is constructed based on *forward-looking* betas, and is therefore not an implementable strategy. In this paper, we show that this results in a strong bias in estimated returns, and that a hedging strategy based on *ex-ante* betas does not exhibit the performance improvement noted in Grundy and Martin (2001), which is not implementable in real time.[5]

The source of the bias is a striking correlation of the loser-portfolio beta with the return on the market. In a bear market, we show that the up- and down-market betas differ substantially for the momentum portfolio. Using a Henriksson and Merton (1981) specification, we calculate up- and down-betas for the momentum portfolios. We show that, in a bear market, a momentum portfolio's up-market beta is more than double its down-market beta (-1.51 versus -0.70), and that this difference is highly statistically significant (t-stat $= 4.5$). Outside of bear markets, there is no statistically significant difference in betas.

[5]The result that the betas of winner-minus-loser portfolios are non-linearly related to contemporaneous market returns has also been documented in Rouwenhorst (1998) who documents this feature for non-US equity momentum strategies (Table V, p. 279). Chan (1988) and DeBondt and Thaler (1987) document this non-linearity for longer-term winner/loser portfolios. However, Boguth, Carlson, Fisher, and Simutin (2010), building on the results of Jagannathan and Korajczyk (1986), note that the interpretation of the measures of abnormal performance (*i.e.*, the alphas) in Chan (1988), Grundy and Martin (2001), and Rouwenhorst (1998) are problematic and provide a critique of Grundy and Martin (2001) and other studies which "overcondition" in a similar way.

More detailed analysis shows that most of the up- versus down-beta asymmetry in bear markets is driven by the past losers, whose up- and down-betas differ by 0.6, while for the past-winner decile the difference is only -0.2. This pattern in dynamic betas of the loser portfolio means that momentum strategies, in bear markets, behave like written calls on the market: When the market falls, they gain a little; when the market rises they lose dramatically.

We next show that these crashes are predictable, and that dynamically hedging market beta exposure *ex-ante* does not significantly improve the returns to momentum strategies. Given the written-call like behavior of the momentum strategy in bear markets, a question that naturally arises given this finding is whether the time variation in the momentum premium we document is related to time-varying exposure to volatility risk.

To examine this hypothesis, we use VIX-imputed variance-swap returns to show that the payoffs to momentum strategies have a strong negative exposure to innovations in market-variance in bear markets (but not in "normal" markets). However, we also show that hedging out this time varying exposure to market-variance (by buying S&P variance swaps in bear markets) does not restore the profitability of momentum in bear markets.

Using the insights from the relationship between momentum payoffs and volatility, and the fact that the momentum strategy volatility is itself predictable and is distinct from the predictability in the mean return, we design an optimal dynamic momentum strategy, which at each point in time, is scaled up or down so as to maximize the unconditional Sharpe ratio of the dynamic portfolio. Specifically, given a conditional expected excess return and conditional volatility for the momentum strategy, we first show theoretically that, to maximize the unconditional Sharpe ratio, a dynamic strategy should scale the weight on the long-short strategy so that, at each point in time, the conditional volatility of the scaled strategy is proportional to the conditional Sharpe ratio of the strategy. Then, we use the insights from our analysis on the forecastability of both the momentum premium and momentum volatility to generate the dynamic weights. We find that the optimal dynamic strategy significantly outperforms the standard static momentum strategy, more than doubling its Sharpe ratio. In addition, the dynamic momentum strategy also significantly outperforms constant volatility momentum strategies suggested in the literature (*e.g.*, Barroso and Santa-Clara (2012)). The reason is because the dynamic strategy not only helps smooth the volatility of the momentum portfolio as does the constant volatility approach, but in addition also exploits the strong forecastability of the Sharpe ratio of the momentum strategy, which we uncover in our analysis.

Given the paucity of momentum crashes and the pernicious effects of data mining from an ever-expanding search across studies and in practice for strategies that improve performance, we challenge the robustness of our findings by replicating our results in different sample periods, four different equity markets, and five distinct asset classes. Across different time periods, markets, and asset classes, we find remarkably consistent results. First, in US data the outperformance of the dynamic strategy is shown to be robust in every quarter-century subsample. Second, we show that all momentum strategies seem to suffer from crashes, all of which are driven by the conditional beta and option-like feature of losers. Specifically, the same option-like behavior is present for cross-sectional equity momentum strategies in Europe, Japan, the UK, and for a global equity momentum strategy. In addition, the optionality is a feature of index futures-, commodity- and currency-momentum strategies. Finally, the same dynamic strategy applied to these alternative asset class momentum strategies is remarkably successful in each and every asset class, generating unconditional Sharpe ratios far higher than the static momentum strategies in each asset class. Even in markets where the static momentum strategy has failed to yield positive profits – *e.g.*, in Japan – our dynamic momentum strategy delivers a significantly positive Sharpe ratio. Applied across all markets and asset classes, a dynamic momentum strategy delivers an annualized Sharpe ratio of 1.18, more than four times larger than that of the static momentum strategy applied to US equities over the same period. Thus, our results pose a significantly greater challenge for rational asset pricing models then the static momentum strategy, based on the arguments in Hansen and Jagannathan (1991).

Finally, we consider several possible explanations for the option-like behavior of momentum payoffs. For equity momentum strategies, one possibility is that the optionality arises because, for a firm with debt in its capital structure, a share of common stock is a call option on the underlying firm value (Merton 1990). Particularly in distressed periods where this option-like behavior is manifested, the underlying firm values in the past loser portfolio have generally suffered severe losses, and are therefore potentially much closer to a level where the option convexity would be strong. The past winners, in contrast, would not have suffered the same losses, and would still be "in-the-money." This hypothesis, however, does not seem applicable for the index futures, commodity, and currency strategies we study, which also exhibit strong option-like behavior. In the conclusion, we briefly discuss a behaviorally motivated explanation for this phenomenon, but a fuller understanding of these convex payoffs is an open area for future research.

The layout of the paper is as follows: Section 2 describes the data and portfolio construction and documents the empirical results for momentum crashes in US equities. In Section 3 we examine the performance of an optimal dynamic strategy based on our findings. In Section 4 we investigate whether the anomalous performance of the momentum strategy can be explained by dynamic loadings on other known factors such as those of Fama and French (1993) and volatility risk. Section 5 performs similar analysis on momentum strategies in international equities and in other asset classes. Section 6 speculates about the sources of the premia we observe, discusses areas for future research, and concludes.

2 US Equity Momentum

In this section, we present the results of our analysis of momentum in US common stocks over the 1927 to 2013 time period. We begin with the data description and portfolio construction.

2.1 US Equity Data and Momentum Portfolio Construction

Our principal data source is the Center for Research in Security Prices (CRSP). We construct monthly and daily momentum decile portfolios, where both sets of portfolios are rebalanced only at the end of each month. The universe starts with all firms listed on NYSE, AMEX or NASDAQ as of the formation date. We utilize only the returns of common shares (with CRSP sharecode of 10 or 11). We require that the firm have a valid share price and a valid number of shares as of the formation date, and that there be a minimum of eight valid monthly returns over the past 11 months, skipping the most recent month, which is our formation period. Following convention and CRSP availability, all prices are closing prices, and all returns are from close to close.

To form the momentum portfolios, we begin by calculating ranking period returns for all firms. The ranking period returns are the cumulative returns from close of the last trading day of each month. Firms are ranked on their cumulative return from 12 months before to one month before the formation date (e.g., cumulative returns from month $t-12$ to $t-2$), where, consistent with the literature (Jegadeesh and Titman (1993), Asness (1995), Fama and French (1996)), we use a one month gap between the end of the ranking period and the start of the holding period to avoid the short-term one-month reversals documented by Jegadeesh (1990) and Lehmann (1990).

All firms meeting the data requirements are placed into one of ten decile portfolios on the basis of their cumulative returns over the ranking period. The firms with the highest ranking period returns go into portfolio 10 – the "[W]inner" decile portfolio – and those with the lowest go into portfolio 1, the "[L]oser" decile. We also evaluate the returns for a zero investment Winner-Minus-Loser (WML) portfolio, which is the difference of the Winner and Loser portfolio each period.

The holding period returns of the decile portfolios are the value-weighted returns of the firms in the portfolio over the one month holding period from the closing price of the last trading day of the previous month through the last trading day of the current month. Given the monthly formation process, portfolio membership does not change within a month, except in the case of delisting. This means that except for cash infusions and payouts and delistings, the portfolios are buy and hold portfolios within each month.

The market return is the value weighted index of all firms meeting our data requirements, and the risk free rate series is the one-month Treasury bill rate.[6]

2.2 Momentum Portfolio Performance

Figure 1 presents the cumulative monthly returns over a 60-year period from 1947:01-2006:12 for investments in: (1) the risk-free asset; (2) the market portfolio; (3) the bottom decile "past loser" portfolio; and (4) the top decile "past winner" portfolio. Recall that our market portfolio is the CRSP value-weighted index, and that the loser and winner portfolios are both value weighed. On the right side of the plot, we present the final dollar values for each of the four portfolios, given a $1 investment in January, 1947 (and, of course, assuming no transaction costs).

This particular 60-year period is chosen purposely because it avoids the turbulent great-depression and WW-II periods before 1947, and the great recession after 2007. Also, this period spans the original momentum studies' sample periods. Consistent with the existing

[6] The source of the market return and of the one-month Treasury-bill rate is Ken French's data library. Per the description on Ken French's website, the market return in month t is the value weighted return of all firms incorporated in the US, listed on the NYSE, AMEX, or NASDAQ, having a CRSP share code of 10 or 11 at the beginning of month t, and having a valid number of shares, price and return data for month t. Again per the description on Ken French's website, the source of the one month treasury bill rate is Ibbotson associates. We convert the monthly risk-free rate series to a daily series by converting the risk-free rate at the beginning of each month to a daily rate, and assuming that that daily rate is valid through the month.

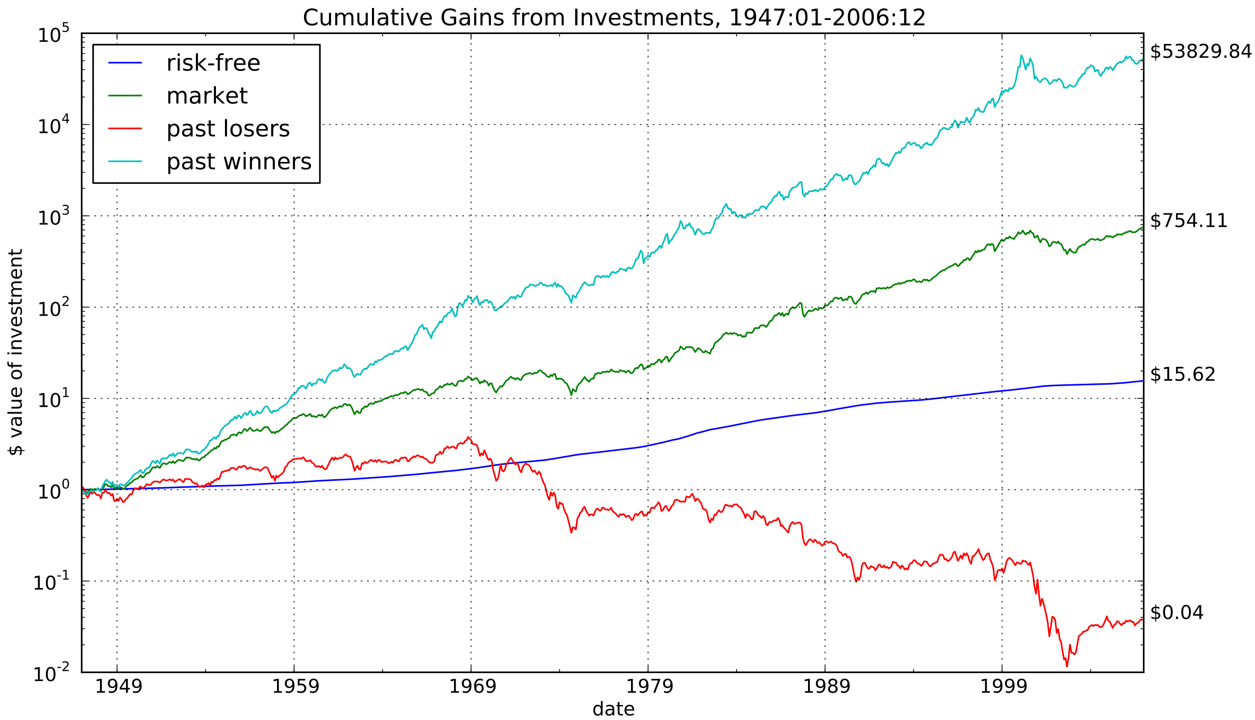

Figure 1: **Momentum Components, 1947-2006**

Over the the 60-year period from 1947:01 through 2006:12, we plot the cumulative returns for four assets: (1) the risk-free asset; (2) the CRSP value-weighted index; (3) the bottom decile "past loser" portfolio; and (4) the top decile "past winner" portfolio. On the right side of the plot, we present the final dollar values for each of the four portfolios, given a $1 investment in January, 1947

literature, there is a strong momentum premium over this 60-year period. Table 1 presents return moments for the momentum decile portfolios over this period. The winner decile excess return averages 16.0% per year, and the loser portfolio averages -6.1% per year. In contrast the average excess market return is 7.5%. The Sharpe ratio of the WML portfolio is 1.08, and that of the market is 0.52. Over this period, the beta of the WML portfolio is slightly negative, -0.25, giving it an unconditional CAPM alpha of 28.6% per year (t-stat = 9.0). As one would expect given the high alpha, an *ex-post* optimal combination of the market and WML portfolios has a Sharpe ratio more than double that of the market. A pattern that we will explore further is the skewness of these portfolios. The winner portfolios are considerably more negatively skewed (monthly and daily) than the loser portfolios, even over this relatively benign period.

Table 1: **Momentum Portfolio Characteristics, 1947:01-2006:12**

This table presents characteristics of the monthly momentum portfolio excess returns over the 60 year period from 1947:01-2006:12. The mean return, standard deviation, and alpha are in percent, and annualized. The Sharpe ratio is annualized. The α, $t(\alpha)$, and β are estimated from a full-period regression of each decile portfolio's excess return on the excess CRSP-value weighted index. For all portfolios except WML, SK denotes the full-period realized skewness of the monthly log returns (not excess) to the portfolios. For WML, SK is the realized skewness of $\log(1+r_{\text{WML}}+r_f)$.

	Momentum Decile Portfolios											
	1	2	3	4	5	6	7	8	9	10	WML	Mkt
$\overline{r-r_f}$	-6.1	1.5	2.8	5.5	6.7	6.5	8.4	10.2	11.5	16.0	26.7	7.5
σ	28.2	22.0	18.7	16.4	15.2	14.7	14.9	15.5	17.1	21.9	24.6	14.5
α	-17.2	-7.7	-5.4	-1.9	-0.5	-0.6	1.2	2.8	3.6	6.7	28.6	0
$t(\alpha)$	(-7.2)	(-4.5)	(-4.1)	(-1.9)	(-0.5)	(-0.8)	(1.7)	(3.5)	(3.7)	(4.1)	(9.0)	(0)
β	1.48	1.22	1.08	0.99	0.95	0.94	0.96	0.98	1.05	1.23	-0.25	1
SR	-0.22	0.07	0.15	0.34	0.44	0.44	0.57	0.66	0.68	0.73	1.08	0.52
SK(m)	-0.29	-0.35	-0.37	-0.19	-0.58	-0.61	-0.59	-0.57	-0.80	-0.62	-2.01	-0.71
SK(d)	-0.23	-0.06	0.03	-0.24	-0.41	-0.93	-1.19	-1.26	-0.86	-0.80	-0.60	-0.71

2.3 Momentum Crashes

Since 1926, however, there have been a number of long periods over which momentum underperformed dramatically. Figure 2 plots the cumulative (monthly) log returns to an investment in the WML portfolio over the entire sample period from 1927:01 to 2013:03.[7] As the figure shows, there are several momentum "crashes," despite the fact that the momentum strategy generates substantial profits over time.

Zeroing in on these crash periods, Figures 3 and 4 show the cumulative daily returns to the same set of portfolios from Figure 1—risk-free, market, past losers, past winners—from March 8, 2009 through March 28, 2013, and over a period starting in June 1932 and continuing through December 1939. These two periods represent the two largest and sustained drawdown periods for the momentum strategy and are selected purposely to illustrate the crashes we study more generally in this paper. As both figures indicate, over both of these periods, the loser portfolio strongly outperforms the winner portfolio. From March 8, 2009 to March 28, 2013, the losers produce more than twice the profits as the winners, which also underperform the market over this period. From June 1, 1932 to December 30, 1939 the losers outperform the winners by 50 percent.

Table 2 presents sample return moments for the momentum decile portfolios over the full

[7] The calculation of cumulative returns for long-short portfolios is described in Appendix A.1.

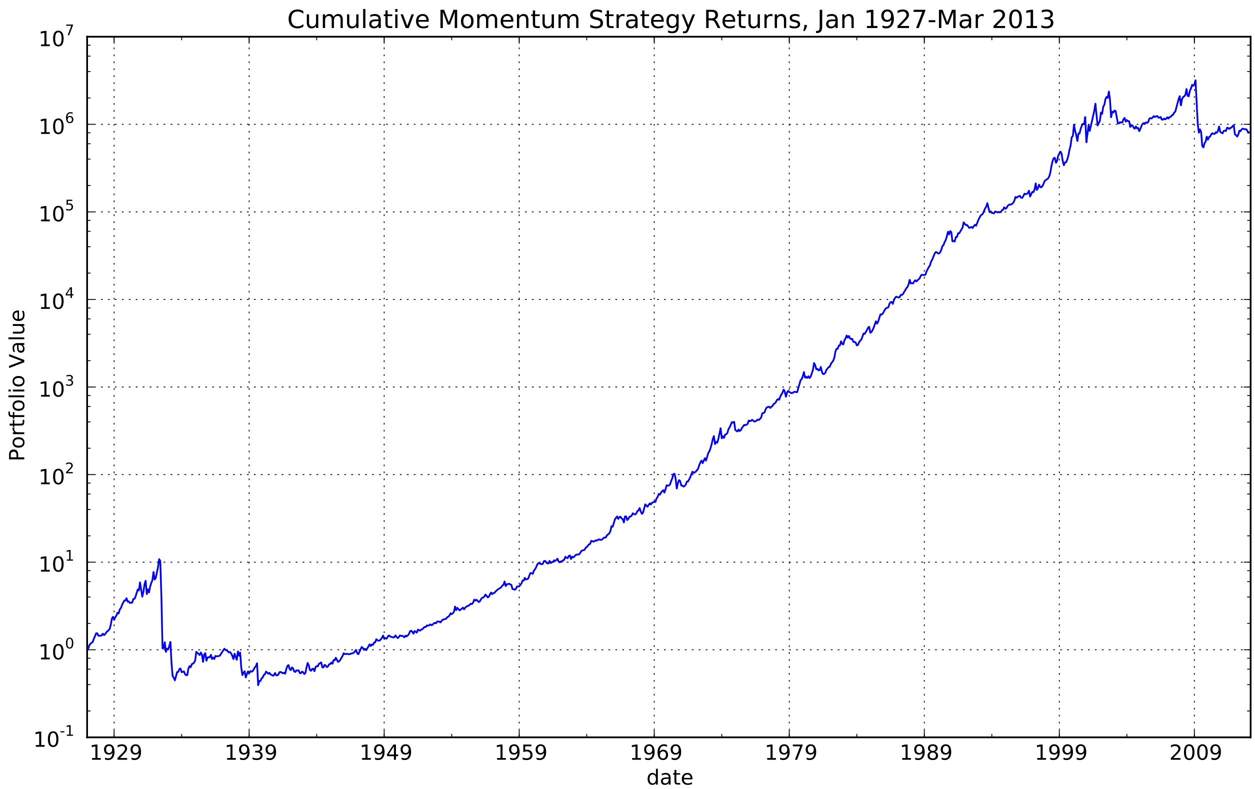

Figure 2: **Cumulative Momentum Returns**
This plot shows the value of an investment in the WML portfolio over the full sample period of 1927:01 to 2013:03, calculated as described in Appendix A.1

sample period from January 1927 to March 2013 that includes these momentum crash periods. While the winners still outperform the losers over time, a WML portfolio delivers a smaller positive Sharpe ratio and market alpha due to these crash episodes. However, the Sharpe ratio and alpha understate the significance of these crashes. Looking at the skewness of the portfolios, we see that the winners become more negatively skewed as we move to more extreme deciles. For the top winner decile portfolio, the monthly (daily) skewness is -0.82 (-0.61), while for the most extreme bottom decile losers the skewness is 0.09 (0.12). The WML portfolio over this full sample period has a monthly (daily) skewness of -4.70 (-1.18), compared to only -2.01 (-0.60) over the 1947 to 2007 period that did not contain these crashes.

Table 3 presents the worst monthly returns to the WML strategy, as well as the lagged two-year returns on the market, and the contemporaneous monthly market return. Several key points emerge from Table 3 as well as from Figures 2 through 4 that we will examine more formally in the remainder of the paper:

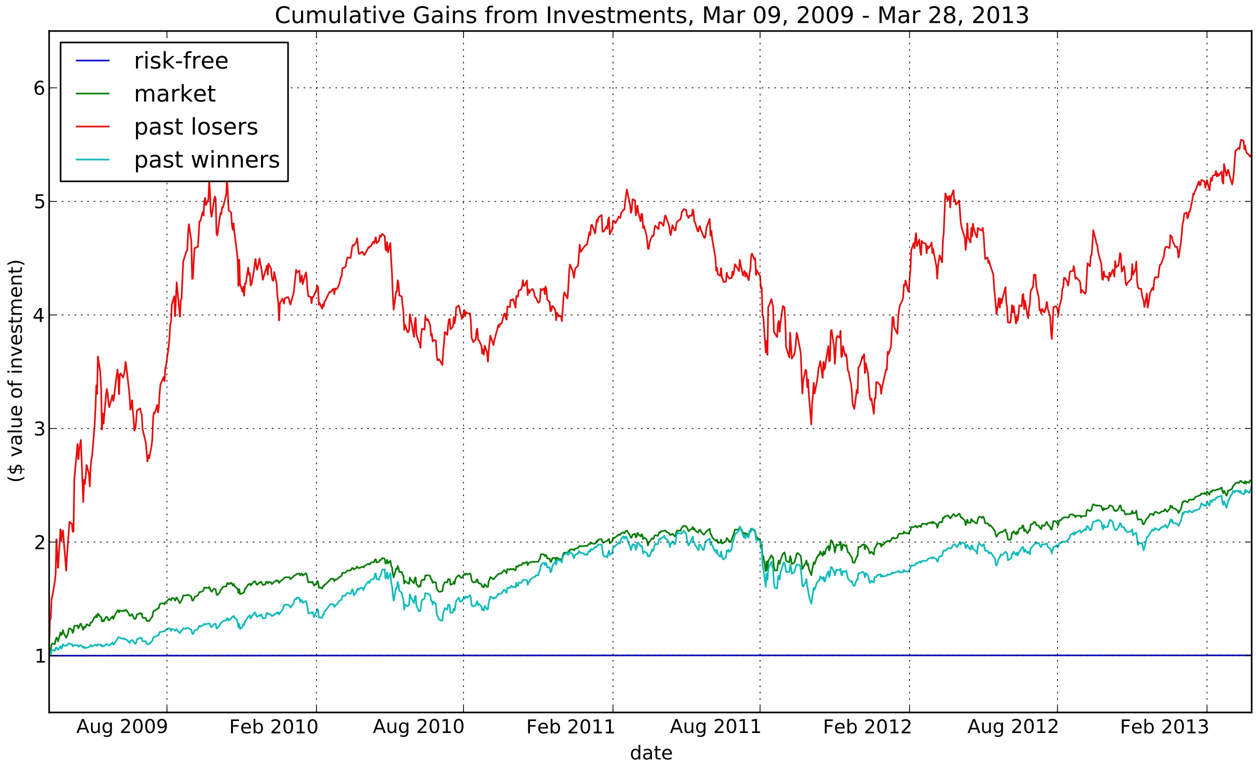

Figure 3: **Momentum Following the 2008-09 Financial Crisis**
Plotted are the cumulative daily returns to the same set of portfolios from Figure 1 from March 8, 2009 through March, 28 2013.

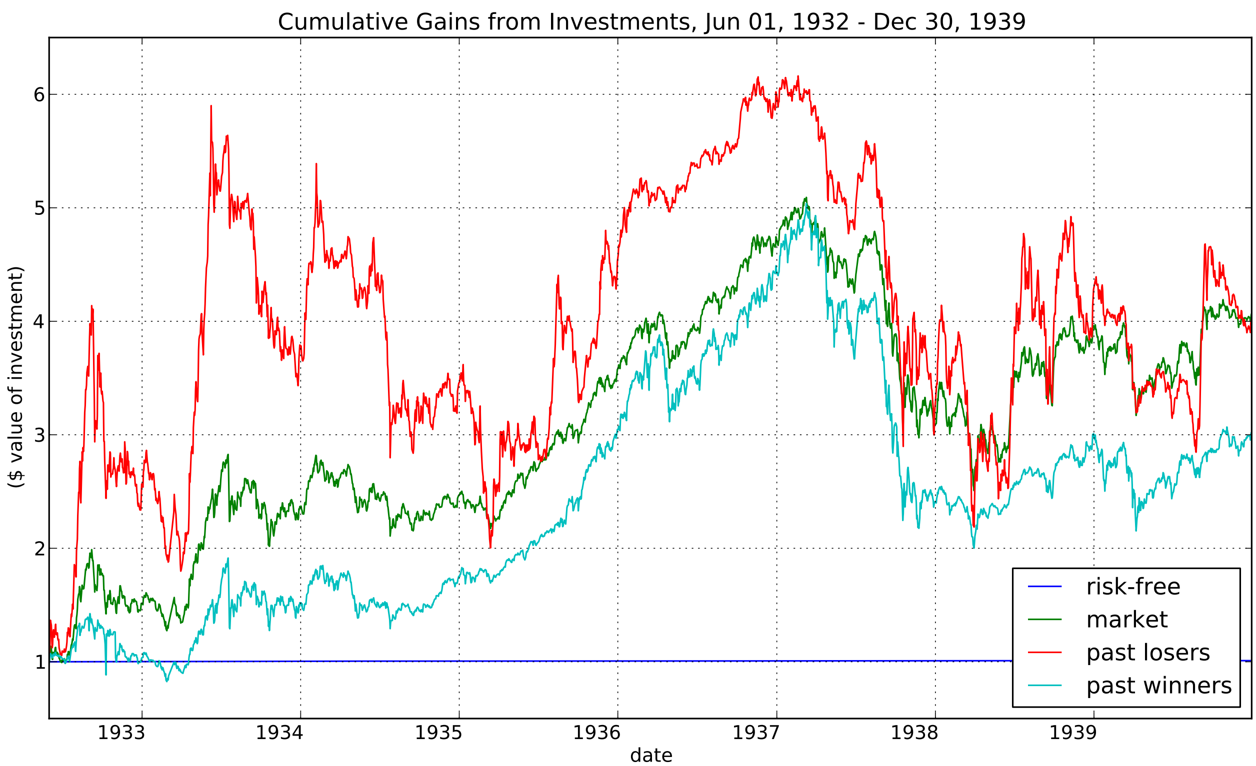

Figure 4: **Momentum During the Great Depression**
Plotted are the cumulative daily returns to the same set of portfolios from Figure 1 from June, 1932 through December 1939.

Table 2: **Momentum Portfolio Characteristics, 1927:01-2013:03**

The calculations for this table are similar those in Table 1, except that the time period is 1927:01-2013:03. SK(m) is the skewness of the monthly log returns, and SK(d) is the skewness of the daily log returns.

	Momentum Decile Portfolios											
	1	2	3	4	5	6	7	8	9	10	WML	Mkt
$\overline{r - r_f}$	-2.5	2.9	2.9	6.4	7.1	7.1	9.2	10.4	11.3	15.3	21.3	7.6
σ	36.5	30.5	25.9	23.2	21.3	20.2	19.5	19.0	20.3	23.7	30.1	18.8
α	-14.8	-7.8	-6.5	-2.2	-0.9	-0.7	1.7	3.2	3.8	7.5	25.7	0
$t(\alpha)$	(-6.7)	(-4.8)	(-5.3)	(-2.2)	(-1.1)	(-1.0)	(2.7)	(4.5)	(4.3)	(5.1)	(8.5)	(0)
β	1.61	1.41	1.24	1.13	1.06	1.03	0.99	0.95	0.99	1.04	-0.58	1
SR	-0.07	0.09	0.11	0.28	0.33	0.35	0.47	0.54	0.56	0.65	0.71	0.40
SK(m)	0.09	-0.05	-0.19	0.21	-0.13	-0.30	-0.55	-0.54	-0.76	-0.82	-4.70	-0.56
SK(d)	0.12	0.29	0.22	0.27	0.10	-0.10	-0.44	-0.66	-0.67	-0.61	-1.18	-0.56

1. While past winners have generally outperformed past losers, there are relatively long periods over which momentum experiences severe losses or "crashes."

2. Every one of the 15 worst momentum returns occurs when the lagged two-year market return is negative. The worst 14 occur in months where the market rose, often in a dramatic fashion.

3. The clustering evident in this table, and the daily cumulative returns in Figure 3 and 4, make it clear that the crashes have relatively long duration. They do not occur over the span of minutes or days—a crash is not a Poisson jump. They take place slowly, over the span of multiple months.

4. Similarly, the extreme losses are clustered: The two worst months for momentum are back-to-back, in July and August of 1932, following a market decline of roughly 90% from the 1929 peak. March and April of 2009 are the 7th and 4th worst momentum months, respectively, and April and May of 1933 are the 6th and 12th worst. Three of the ten worst momentum monthly returns are from 2009—over a three-month period in which the market rose dramatically and volatility fell. While it might not seem surprising that the most extreme returns occur in periods of high volatility, the effect is asymmetric for losses versus gains: the extreme momentum gains are not nearly as large in magnitude, or as concentrated in time.

5. Closer examination reveals that the crash performance is mostly attributable to the short side or the performance of losers. For example, in July and August of 1932, the market actually rose by 82%. Over these two months, the winner decile rose by 30%, but the loser decile was up by 236%. Similarly, over the three month period from March to May of 2009, the market was up by 29%, but the loser decile was up by 156%. Thus, to the extent that the strong momentum reversals we observe in the data can be characterized as a crash, they are a crash where the short side of the portfolio—the losers—are "crashing up" rather than down.

Table 3: **Worst Monthly Momentum Returns**

This table presents the 15 worst monthly returns to the WML portfolio over the 1927:01-2013:03 time period. Also tabulated are MKT-2Y, the 2-year market returns leading up to the portfolio formation date, and MKT_t, the market return in the same month. The dates in black are between 1932-07 and 1939-09; blue are between April and August of 2009; and red are from January 2001 and November 2002. Green is the remainder.

Rank	Month	WML_t	Mkt-2y	Mkt_t
1	1932-08	-74.36	-69.39	36.49
2	1932-07	-60.98	-76.22	33.48
3	2001-01	-49.19	-9.95	2.58
4	2009-04	-45.52	-46.33	10.18
5	1939-09	-43.83	-21.34	16.64
6	1933-04	-43.14	-60.33	37.67
7	2009-03	-42.28	-50.61	8.93
8	2002-11	-37.04	-43.85	5.84
9	1938-06	-33.36	-28.29	23.69
10	2009-08	-30.54	-32.15	3.31
11	1931-06	-29.72	-53.25	13.61
12	1933-05	-28.90	-39.39	21.26
13	2001-11	-25.31	-34.50	7.37
14	2001-10	-24.98	-32.27	2.25
15	1974-01	-24.04	-23.71	-0.80

2.4 Risk of Momentum Returns

Table 3 also suggests that large changes in market beta may help to explain some of the large negative returns earned by momentum strategies. For example, as of the beginning of March 2009, the firms in the loser decile portfolio were, on average, down from their peak by 84%. These firms included the firms that were hit hardest in the financial crisis: among them Citigroup, Bank of America, Ford, GM, and International Paper (which was levered). In contrast, the past-winner portfolio was composed of defensive or counter-cyclical firms like Autozone. The loser firms, in particular, were often extremely levered, and at risk of bankruptcy. In the sense of the Merton (1990) model, their common stock was effectively an out-of-the-money option on the underlying firm value. This suggests that there were potentially large differences in the market betas of the winner and loser portfolios.

To investigate the time-varying betas of winners and losers, Figure 5 plots the market betas for the winner and loser momentum deciles, estimated using 126 day ($\approx$ 6 month) rolling

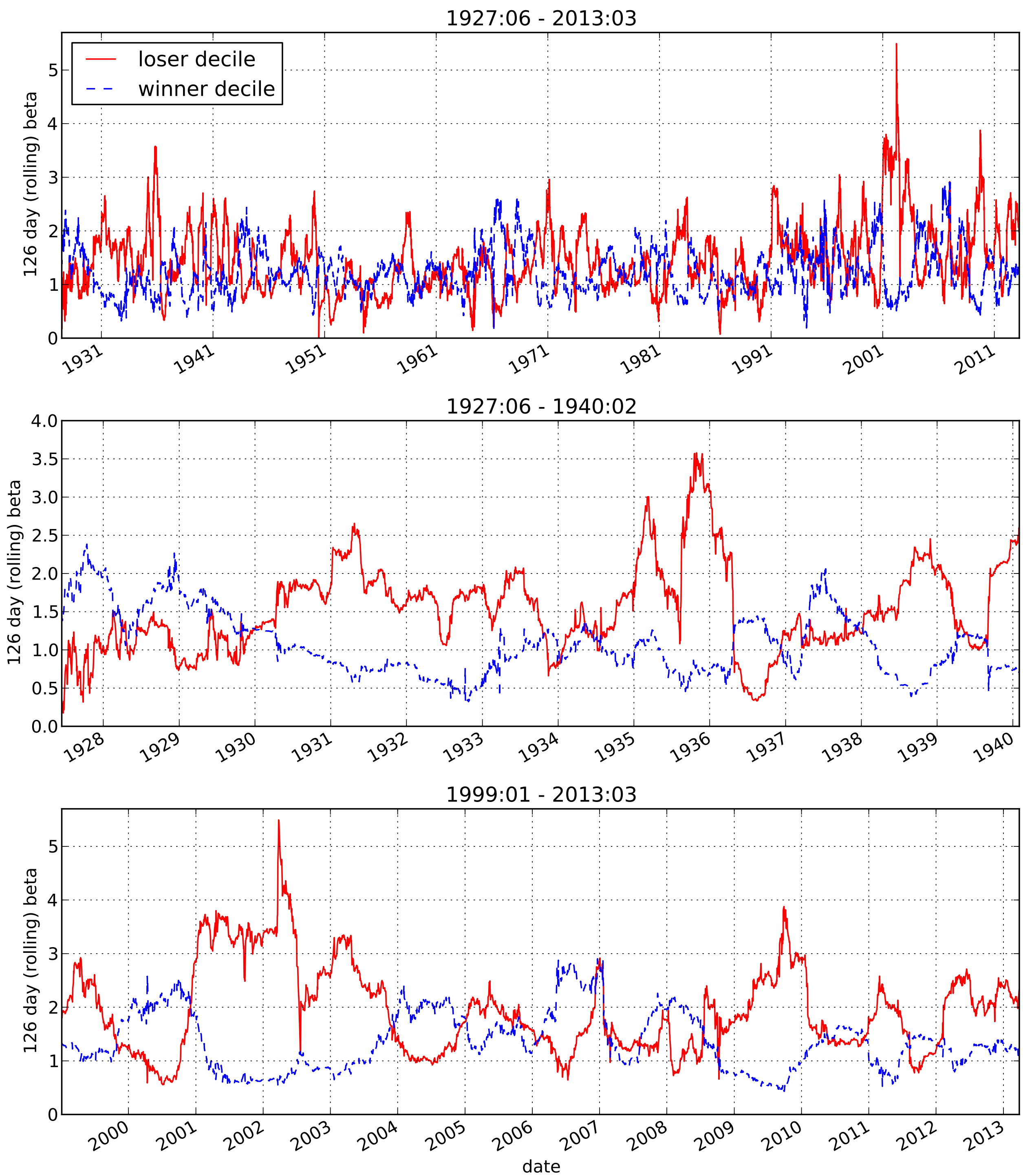

Figure 5: **Market Betas of Winner and Loser Decile Portfolios**

These three plots present the estimated market betas over our full sample, and over the periods 1931-1940 and 1999-2013. The betas are estimated by running a set of 126-day rolling regressions. Each regression uses 10 (daily) lagged market returns in the estimations of the beta as a way of accounting for the lead-lag effects in the data.

market model regressions with daily data.[8]

Figure 5 presents three plots of these time-varying betas. The first plots the rolling six-month betas over the full sample period from June 1927 to March 2013. As the plot shows, the betas move around substantially, especially for the losers portfolio, whose beta tends to increase dramatically during volatile periods. The second and third plots zoom in on these crash episodes by plotting the betas several years before, during, and after the crashes, from 1927-1940 and from 1999-2013, respectively. These plots both show that in volatile market times betas of the winner and loser portfolios move above and below one (generally, the average is close to, but a bit above, one as expected). The beta of the winner portfolio is sometimes above 2 following large market rises. However, for the loser portfolio, the betas reach far higher levels. The widening beta differences between winners and losers, coupled with the facts from Table 3 that these crash periods are characterized by sudden and dramatic market upswings, means that the WML strategy will experience huge losses. In the next few subsections, we will examine these patterns more formally by characterizing the beta variation and investigating how the apparent variation in the mean return of the momentum portfolio is linked to the time variation in market risk.

2.5 Hedging the Market Risk in the Momentum Portfolio

To begin, we note that Grundy and Martin (2001) explore this same question, and argue that the poor performance of the momentum portfolio in the pre-WWII period noted by Jegadeesh and Titman (1993) is a result of the time varying market and size beta. Specifically, they argue a hedged momentum portfolio – for which conditional market and size exposure is zero – has a high average return and a high Sharpe-ratio in the pre-WWII period when the unhedged momentum portfolio suffers.

At the time that Grundy and Martin (2001) undertook their research, daily stock return

[8] We use 10 daily lags of the market return in estimating the market betas. Specifically, we estimate a daily regression specification of the form:

$$\tilde{r}^e_{i,t} = \beta_0 \tilde{r}^e_{m,t} + \beta_1 \tilde{r}^e_{m,t-1} + \cdots + \beta_{10} \tilde{r}^e_{m,t-10} + \tilde{\epsilon}_{i,t}$$

and then report the sum of the estimated coefficients $\hat{\beta}_0 + \hat{\beta}_1 + \cdots + \hat{\beta}_{10}$. Particularly for the past loser portfolios, and especially in the pre-WWII period, the lagged coefficients are strongly significant, suggesting that market wide information is incorporated into the prices of many of the firms in these portfolios over the span of multiple days. See Lo and MacKinlay (1990) and Jegadeesh and Titman (1995).

data was not available through CRSP in the pre-1962 period. Given the dynamic nature of momentum's risk-exposures, estimating the future hedge coefficients *ex-ante* is problematic. As a result they investigate the efficacy of hedging primarily based on an *ex-post* estimate of the portfolio's market and size betas, estimated using monthly returns over the current month and the future five months.

However, to the extent that the future momentum-portfolio beta is correlated with the future return of the market, this procedure will result in a biased estimate of the returns of the hedged portfolio. Boguth, Carlson, Fisher, and Simutin (2010) critique the Grundy and Martin (2001) test, and argue that this "overconditioning" can lead to biased results. In Section 2.6, we will show there is in fact a strong correlation of this type which indeed results in a large upward bias in the estimated performance of the hedged portfolio.

We first estimate the performance of a WML portfolio which hedges out market risk using an *ex-post* estimate of market beta, following Grundy and Martin (2001). We construct the *ex-post* hedged portfolio in a similar way, though using daily data. Specifically, the size of the market hedge is based on the future 42-day (2 month) realized market beta of the portfolio being hedged. Again, to calculate the beta we use 10 daily lags of the market return.

Figure 6 plots the performance of the *ex-post* hedged WML portfolio over the period 1928 to 1945, and that of the unhedged portfolio. The ex-post hedged portfolio exhibits considerably improved performance, consistent with the results of Grundy and Martin (2001).

2.6 Option-like Behavior of the WML portfolio

We show that the strong realized performance of the *ex-post* hedged portfolio is an upward biased estimate of the *ex-ante* performance of the portfolio. The source of the bias is that in down markets, the market beta of the WML portfolio is strongly *negatively* correlated with the contemporaneous realized performance of the portfolio. This means that the *ex-post* hedge will have a higher market beta when future market returns are high, and a lower beta when future market returns are low.

We also show that the return of the momentum portfolio, net of market risk, *is* significantly lower in bear markets. Both of these results are linked to the fact that, in bear markets, the momentum strategy returns behave as if the momentum portfolio is effectively short a call

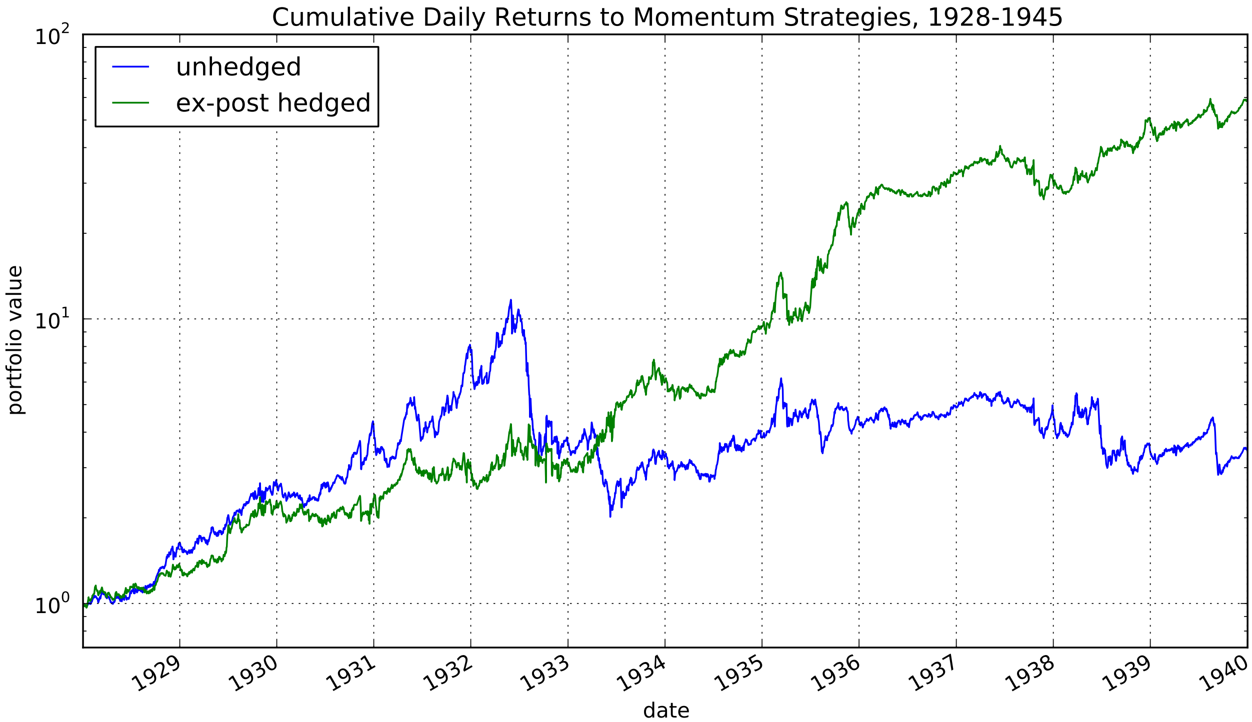

Figure 6: **Ex-post Hedged Momentum Portfolio Performance**
This figure presents the cumulative returns to the baseline WML strategy as well as the WML strategy that conditionally hedges the market exposure using the *ex-post* hedged procedure of Grundy and Martin (2001), but using daily data. Specifically, the size of the market hedge is based on the *future* 42-day (2 month) realized market beta of the WML portfolio.

option on the market.

We first illustrate these issues with a set of monthly time-series regressions, the results of which are presented in Table 4. The variables used in the regressions are:

1. $\tilde{R}_{\text{WML},t}$ is the WML return in month t.
2. $\tilde{R}^e_{m,t}$ is the CRSP value-weighted index excess return in month t.
3. $\text{I}_{B,t-1}$ is an *ex-ante* **B***ear-market* indicator that equals 1 if the cumulative CRSP VW index return in the 24 months leading up to the start of month t is *negative*, and is zero otherwise.
4. $\tilde{\text{I}}_{U,t}$ is the contemporaneous – *i.e.,* not *ex-ante* – **U***p-market* indicator variable. It is 1 if the excess CRSP VW index return is greater than the risk-free rate in month t, and is zero otherwise.[9]

[9]Of the 1,035 months in the 1927:01-2013:03 period, there are 183 bear market months by our definition. Also, there are 618 Up-months, and 417 down-months.

Table 4: **Market Timing Regression Results**

This table presents the results of estimating four specifications of a monthly time-series regressions run over the period 1927:01 to 2013:03. In all cases the dependent variable is the return on the WML portfolio. The independent variables are described in Section 2.5.

		Estimated Coefficients *(t-statistics in parentheses)*			
Coeff.	Variable	(1)	(2)	(3)	(4)
$\hat{\alpha}_0$	1	0.019	0.020	0.020	0.020
		(7.3)	(7.7)	(7.8)	(8.4)
$\hat{\alpha}_B$	I_{B-1}		-0.021	0.005	
			(-3.5)	(0.6)	
$\hat{\beta}_0$	$\tilde{R}^e_{m,t}$	-0.577	-0.032	-0.032	-0.034
		(-12.5)	(-0.5)	(-0.5)	(-0.6)
$\hat{\beta}_B$	$\mathrm{I}_{B,t-1} \cdot \tilde{R}^e_{m,t}$		-1.136	-0.668	-0.710
			(-13.4)	(-5.0)	(-6.2)
$\hat{\beta}_{B,U}$	$\mathrm{I}_{B,t-1} \cdot \mathrm{I}_{U,t} \cdot \tilde{R}^e_{m,t}$			-0.810	-0.734
				(-4.5)	(-5.7)
R^2_{adj}		0.130	0.271	0.284	0.285

Regression (1) in Table 4 fits an unconditional market model to the WML portfolio:

$$\tilde{R}_{\mathrm{WML},t} = \alpha_0 + \beta_0 \tilde{R}_{m,t} + \tilde{\epsilon}_t$$

Consistent with the results in the literature, the estimated market beta is somewhat negative, -0.577, and the intercept, $\hat{\alpha}$, is both economically large (1.9% per month), and statistically significant (t-stat = 7.3).

Regression (2) in Table 4 fits a conditional CAPM with the bear market I_B indicator as an instrument:

$$\tilde{R}_{\mathrm{WML},t} = (\alpha_0 + \alpha_{B,t-1} I_{B,t-1}) + (\beta_0 + \beta_B I_{B,t-1}) \tilde{R}_{m,t} + \tilde{\epsilon}_t. \qquad (1)$$

This specification is an attempt to capture both expected return and market-beta differences in bear markets. First, consistent with Grundy and Martin (2001), we see a striking change in the market beta of the WML portfolio in bear markets: it is -1.136 lower, with a t-statistic of -13.4 on the difference. The intercept is also lower: the point estimate for the alpha in bear markets – equal to $\hat{\alpha}_0 + \hat{\alpha}_B$ – is now -1 basis point per month.

Regression (3) introduces an additional element to the regression which allows us to assess the

Table 5: **Momentum Portfolio Optionality in Bear Markets**

This table presents the results of a regressions of the excess returns of the 10 momentum portfolios and the Winner-Minus-Loser (WML) long-short portfolio on the CRSP value-weighted excess market returns, and a number of indicator variables. For each of these portfolios, the regression estimated here is:

$$\tilde{R}^e_{i,t} = [\alpha_0 + \alpha_B I_{B,t-1}] + [\beta_0 + I_{B,t-1}(\beta_B + \tilde{I}_{U,t}\beta_{B,U})]\tilde{R}_{m,t} + \tilde{\epsilon}_t$$

where R_m is the CRSP value-weighted excess market return, $I_{B,t-1}$ is an *ex-ante* **B**ear-market indicator and $I_{U,t}$ is a contemporaneous *UP*-market indicator, as defined in the text on page 17. The time period is 1927:01-2013:03.

Coef.	Momentum Decile Portfolios – Monthly Excess Returns *(t-statistics in parentheses)*										
Est.	1	2	3	4	5	6	7	8	9	10	WML
$\hat{\alpha}_0$	-0.014	-0.008	-0.005	-0.002	-0.001	-0.000	0.002	0.003	0.003	0.006	0.020
	(-7.3)	(-5.7)	(-4.9)	(-2.4)	(-0.7)	(-0.9)	(2.7)	(4.1)	(3.8)	(4.6)	(7.8)
$\hat{\alpha}_B$	-0.002	0.004	-0.001	-0.006	-0.003	-0.002	-0.000	-0.001	0.004	0.003	0.005
	(-0.3)	(0.9)	(-0.4)	(-2.0)	(-1.4)	(-1.2)	(-0.1)	(-0.3)	(1.6)	(0.8)	(0.6)
$\hat{\beta}_0$	1.339	1.153	1.014	0.956	0.924	0.953	0.975	1.020	1.114	1.307	-0.032
	(30.4)	(35.8)	(42.7)	(49.5)	(55.7)	(72.2)	(72.4)	(70.0)	(62.7)	(46.2)	(-0.5)
$\hat{\beta}_B$	0.229	0.332	0.361	0.158	0.178	0.082	0.026	-0.128	-0.157	-0.440	-0.668
	(2.3)	(4.5)	(6.6)	(3.6)	(4.7)	(2.7)	(0.9)	(-3.8)	(-3.9)	(-6.8)	(-5.0)
$\hat{\beta}_{B,U}$	0.596	0.346	0.176	0.350	0.165	0.121	-0.011	-0.029	-0.183	-0.214	-0.810
	(4.4)	(3.5)	(2.4)	(5.9)	(3.2)	(3.0)	(-0.3)	(-0.6)	(-3.3)	(-2.4)	(-4.5)

extent to which the up- and down-market betas of the WML portfolio differ. The specification is similar to that used by Henriksson and Merton (1981) to assess market timing ability of fund managers:

$$\tilde{r}_{\text{WML},t} = [\alpha_0 + \alpha_B \cdot I_{B,t-1}] + [\beta_0 + I_{B,t-1}(\beta_B + \tilde{I}_{U,t}\beta_{B,U})]\tilde{R}_{m,t} + \tilde{\epsilon}_t. \tag{2}$$

If $\beta_{B,U}$ is different from zero, this suggests that the WML portfolio exhibits option-like behavior relative to the market. Specifically, a negative $\beta_{B,U}$ would mean that, in bear markets, the momentum portfolio is effectively short a call option on the market. In months when the contemporaneous market return is negative, the point estimate of the WML portfolio beta is -0.74 ($\hat{\beta}_0 + \hat{\beta}_B$). But, when the market return is positive, the market beta of WML is considerably more negative – specifically, the point estimate is $\hat{\beta}_0 + \hat{\beta}_B + \hat{\beta}_{B,U} = -\mathbf{1.44}$.

The predominant source of this *optionality* turns out to be the loser portfolio. Table 5 presents the results of the regression specification in equation (2) for each of the ten momentum portfolios. The final row of the table (the $\hat{\beta}_{B,U}$ coefficient) shows the strong up-market betas for the loser portfolios in bear markets. For the loser decile, the down-market beta is 1.568

Table 6: **Momentum Portfolio Optionality in Bull Markets**

This table presents the results of a regressions of the excess returns of the 10 momentum portfolios and the Winner-Minus-Loser (WML) long-short portfolio on the CRSP value-weighted excess market returns, and a number of indicator variables. For each of these portfolios, the regression estimated here is:

$$\tilde{R}^e_{i,t} = [\alpha_0 + \alpha_L I_{L,t-1}] + [\beta_0 + I_{L,t-1}(\beta_L + \tilde{I}_{U,t}\beta_{L,U})]\tilde{R}_{m,t} + \tilde{\epsilon}_t$$

where R_m is the CRSP value-weighted excess market return, $I_{L,t-1}$ is an *ex-ante* bul*L*-market indicator and $I_{U,t}$ is a contemporaneous *UP*-market indicator, as defined in the text on page 17. The time period is 1927:01-2013:03.

Coef.	Momentum Decile Portfolios – Excess Monthly Returns *(t-statistics in parentheses)*										
Est.	1	2	3	4	5	6	7	8	9	10	WML
$\hat{\alpha}_0$	-0.001	0.003	-0.003	0.002	0.000	-0.000	0.001	0.001	0.002	0.005	0.006
	(-0.2)	(0.9)	(-1.5)	(1.0)	(0.2)	(-0.0)	(1.1)	(1.1)	(1.3)	(1.9)	(1.1)
$\hat{\alpha}_L$	-0.013	-0.010	-0.002	-0.006	-0.002	-0.000	-0.002	0.001	0.002	0.005	0.019
	(-2.6)	(-2.8)	(-0.8)	(-2.7)	(-1.3)	(-0.2)	(-1.2)	(0.5)	(0.9)	(1.7)	(2.8)
$\hat{\beta}_0$	1.880	1.655	1.453	1.301	1.187	1.095	0.997	0.880	0.861	0.765	-1.115
	(41.1)	(49.3)	(58.9)	(64.4)	(69.3)	(80.4)	(72.8)	(59.1)	(47.0)	(26.3)	(-18.4)
$\hat{\beta}_L$	-0.537	-0.490	-0.446	-0.407	-0.304	-0.138	-0.080	0.127	0.282	0.664	1.201
	(-5.9)	(-7.3)	(-9.0)	(-10.0)	(-8.9)	(-5.0)	(-2.9)	(4.3)	(7.7)	(11.4)	(9.9)
$\hat{\beta}_{L,U}$	0.007	-0.009	0.028	0.141	0.092	-0.002	0.123	0.028	-0.062	-0.272	-0.279
	(0.0)	(-0.1)	(0.4)	(2.3)	(1.7)	(-0.0)	(2.9)	(0.6)	(-1.1)	(-3.0)	(-1.5)

(= 1.339 + 0.229) and the point estimate of the up-market beta is 2.164 (= 1.568 + 0.596). Also, note the slightly negative up-market beta increment for the winner decile (= −0.214). This pattern also holds for less extreme winners and losers, such as decile 2 versus decile 9 or decile 3 versus 8, with the differences between winners and losers declining monotonically for less extreme past-return sorted portfolios.

2.7 Asymmetry in the Optionality

It is interesting that the optionality associated with the loser portfolios that is apparent in the regressions in Table 5 is only present in bear markets. Table 6 presents the same set of regressions as in Table 5, only now instead of using the Bear-market indicator $I_{B,t-1}$, we use the Bul**L** market indicator $I_{L,t-1} = (1 - I_{B,t-1})$. The key variables here are the estimated coefficients and t-statistics on $\beta_{L,U}$, presented in the last two rows of Table 6. Unlike in Table 5, no significant asymmetry is present in the loser portfolio, while the winner portfolio asymmetry is comparable to what is shown in Table 5. For the winner portfolios, we obtain the same slightly negative point estimate for the up-market beta increment. There is no

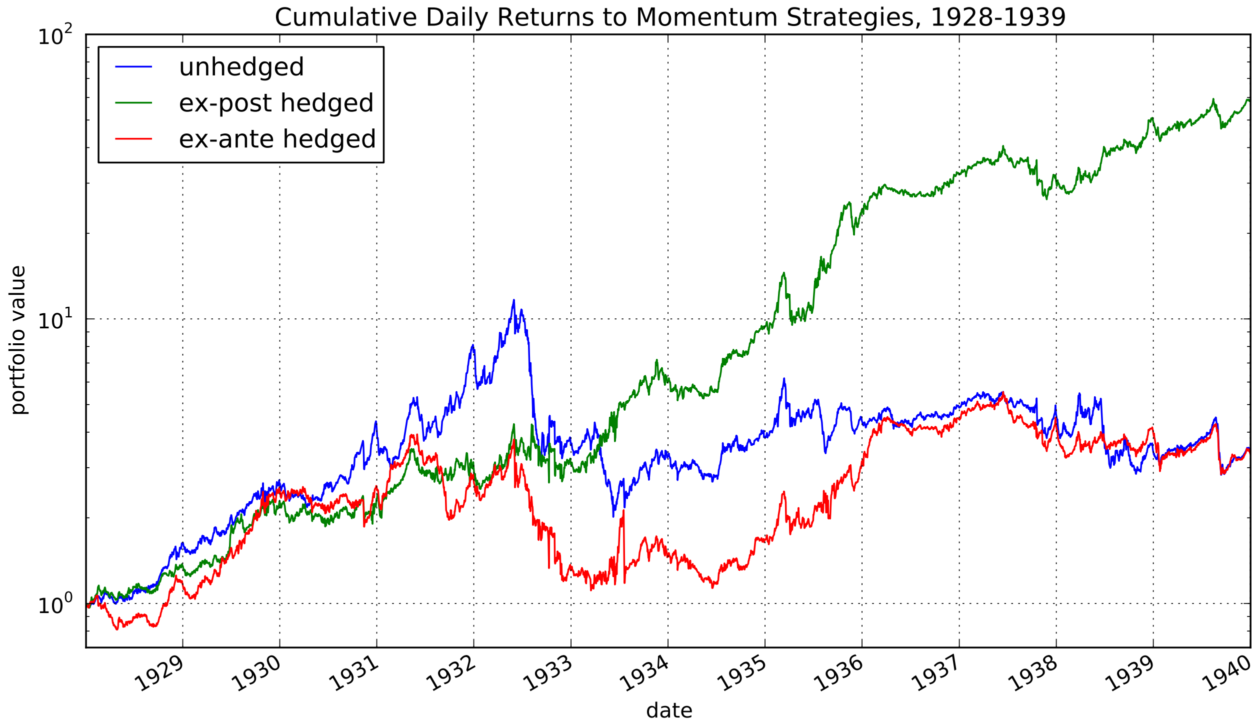

Figure 7: ***Ex-Ante* Hedged Portfolio Performance**

Plotted along with the two series from Figure 6—the static momentum strategy returns and the ex post hedged momentum strategy returns—are the cumulative log return to an *ex-ante* hedged momentum portfolio, where betas are estimated using the lagged 42 days of returns on the portfolio and the market from equation (8).

apparent variation associated with the past market return. Finally, the WML portfolio shows no statistically significant optionality in bull markets, unlike what is seen in bear markets.

2.8 *Ex-ante* Hedge of the market risk in the WML Portfolio

The results of the preceding analysis suggest that calculating hedge ratios based on future realized betas, as in Grundy and Martin (2001), is likely to produce strongly upward biased estimates of the performance of the hedged portfolio. As we have seen, the realized market beta of the momentum portfolio tends to be more negative when the realized return of the market is positive. Thus, the hedged portfolio – where the hedge is based on the future realized portfolio beta – buys the market (as a hedge) when the future market return is high, leading to a strong upward bias in the estimated performance of the *ex-post* hedged portfolio.

Figure 7 adds the cumulative log return to an *ex-ante* hedged momentum portfolio, where betas are estimated using the lagged 42 days of returns of the portfolio on the market. The strong bias in the *ex-post* hedge is clear here, as the *ex ante* hedged portfolio performs no

Table 7: **Momentum Returns and Estimated Market Variance**

Each column of this table presents the estimated coefficients and t-statistics for a time-series regression based on the following regression specification:

$$\tilde{r}_{\mathrm{WML},t} = \gamma_0 + \gamma_{\mathrm{B}} \cdot \mathrm{I}_{B,t-1} + \gamma_{\sigma^2_m} \cdot \hat{\sigma}^2_{m,t-1} + \gamma_{int} \cdot \mathrm{I}_{\mathrm{B,t-1}} \cdot \hat{\sigma}^2_{m,t-1} + \tilde{\epsilon}_t$$

Here, $\mathrm{I}_{B,t-1}$ is the bear market indicator described on page 17. $\hat{\sigma}^2_{m,t-1}$ is the variance of the daily returns on the market, measured over the 126-days preceding the start of month t. The regression is monthly, over the period 1927:07-2013:03.

	(1)	(2)	(3)	(4)	(5)
$\hat{\gamma}_0$	0.020	0.036	0.033	0.021	0.022
	(6.6)	(6.6)	(6.0)	(7.1)	(3.3)
$\hat{\gamma}_B$	-0.027		-0.014		0.025
	(-3.8)		(-1.8)		(1.5)
$\hat{\gamma}_{\sigma^2_m}$		-0.009	-0.007		-0.001
		(-4.4)	(-2.9)		(-0.5)
$\hat{\gamma}_{int}$				-0.009	-0.013
				(-5.2)	(-2.8)

better than the unhedged WML portfolio in the overall 1928-1939 period.

2.9 Market Stress and Momentum Returns

One very casual interpretation of the results presented in Section 2.6 is that there are option like payoffs associated with the past losers in bear markets, and that the value of this option is not adequately reflected in the prices of past losers. This interpretation further suggests that the value of this option should be a function of the future variance of the market.

In this section we examine this hypothesis. Using daily market return data, we construct an *ex-ante* estimate of the market volatility over the next one month. In Table 7, we use this market variance estimate in combination with the bear-market indicator I_{B} previously employed to forecast future WML returns. Specifically, we run the following regression:

$$\tilde{r}_{\mathrm{WML},t} = \gamma_0 + \gamma_{\mathrm{B}} \cdot \mathrm{I}_{\mathrm{B}} + \gamma_{\sigma^2_m} \cdot \hat{\sigma}^2_{m,t-1} + \gamma_{int} \cdot \mathrm{I}_{\mathrm{B}} \cdot \hat{\sigma}^2_{m,t-1} + \tilde{\epsilon}_t \qquad (3)$$

where I_{B} is the bear market indicator described on page 17 and $\hat{\sigma}^2_{m,t-1}$ is the variance of the daily returns of the market over the 126-days just prior to the start of month t.

Table 7 shows that both estimated market variance and the bear market indicator independently forecast future momentum returns. Columns (1) and (2) report regression results for each market stress variable—the bear market indicator and volatility—separately and column (3) reports results using both variables simultaneously. The direction is as suggested by the results of the previous section: in periods of high market stress, as indicated by bear markets and high volatility, momentum returns are low. Finally, the last two columns of Table 7 report results for the interaction between the bear market indicator and volatility, where momentum returns are shown to be particularly poor during bear markets with high volatility.

3 Dynamic Weighting of the Momentum Portfolio

Using the insights from the previous section, we next evaluate the performance of a strategy which dynamically adjusts the weight on the basic WML strategy using the forecasted return and variance of the WML strategy. We show that the dynamic strategy generates a Sharpe ratio more than double that of the baseline \$1-long/\$1-short WML strategy of the sort typically utilized in academic studies, and which we have so far employed in this paper.

We begin with the design of a strategy which dynamically weights WML depending on its forecasted return and volatility. We show in Appendix B that, for the objective function of maximizing the in-sample unconditional Sharpe ratio, the optimal weight on the risky asset (WML) at time $t-1$ is:

$$w^*_{t-1} = \left(\frac{1}{2\lambda}\right)\frac{\mu_{t-1}}{\sigma^2_{t-1}} \tag{4}$$

where $\mu_{t-1} \equiv \mathbb{E}_{t-1}[r_{\mathrm{WML},t}]$ is the conditional expected return on the (zero-investment) WML portfolio over the coming month, $\sigma^2_{t-1} \equiv \mathbb{E}_{t-1}[(r^2_{\mathrm{WML},t} - \mu_{t-1})^2]$ is the conditional variance of the WML portfolio return over the coming month, and λ is a time-invariant scalar that controls the unconditional risk and return of the dynamic portfolio.

We use the insights from our previous analysis to provide an estimate of μ_t, the conditional mean return of WML. The results from Table 7 provide us with an instrument for the time t conditional expected return on the WML portfolio. As a proxy for the expected return, we use the interaction between the bear-market indicator $I_{B,t-1}$ and the market variance over the preceding 6-months as estimated in the last column of Table 7.

Then, to estimate the volatility of the WML series, we first fit a GARCH model as proposed by Glosten, Jagannathan, and Runkle (1993, GJR) to the WML return series. The process is defined by:

$$r_{WML,t} = \mu + \epsilon_t, \tag{5}$$

where $\epsilon_t \sim \mathcal{N}(0, \sigma_t^2)$ and where the evolution of σ_t^2 is governed by the process:

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + (\alpha + \gamma I(\epsilon_{t-1} < 0))\,\epsilon_{t-1}^2 \tag{6}$$

where $I(\epsilon_{t-1} < 0)$ is an indicator variable which is one if $\epsilon_{t-1} < 0$, and zero otherwise.[10] We use maximum likelihood to estimate the parameter set $(\mu, \omega, \alpha, \gamma, \beta)$ over the full time series. Maximum likelihood estimates of the parameters and standard errors are given in Appendix C.

We then form a linear combination of the forecast of future volatility from the fitted GJR-GARCH process with the realized standard deviation of the 126 daily returns preceding the current month. We show in Appendix C that both components contribute to forecasting future daily realized WML volatility.

Our analysis in this section is also related to work by Barroso and Santa-Clara (2012), who argue that momentum crashes can be avoided with a momentum portfolio which is scaled by the trailing volatility of the momentum portfolio. They further show that the unconditional Sharpe ratio of the constant-volatility momentum strategy is far better than the $1-long/$1-short strategy typically used in academic studies.

Equation (4) shows that our results would be approximately the same as those of Barroso and Santa-Clara (2012) if it were the case that the Sharpe ratio of the momentum strategy were time-invariant, *i.e.*, that the forecast mean was always proportional to the forecast volatility. If this were the case then the conditional Sharpe ratio would be equal to the unconditional Sharpe ratio, and the optimal dynamic strategy would be a constant volatility strategy like the one proposed by Barroso and Santa-Clara (2012).

However, this is not the case for momentum. In fact, the return of WML is slightly *negatively* related to the forecast of WML return volatility. This means that the volatility of the optimal

[10] Engle and Ng (1993) investigate the performance of a number of parametric models in explaining daily market volatility for Japan. They find that the GJR model that we use here best fits the dynamic structure of volatility for that market.

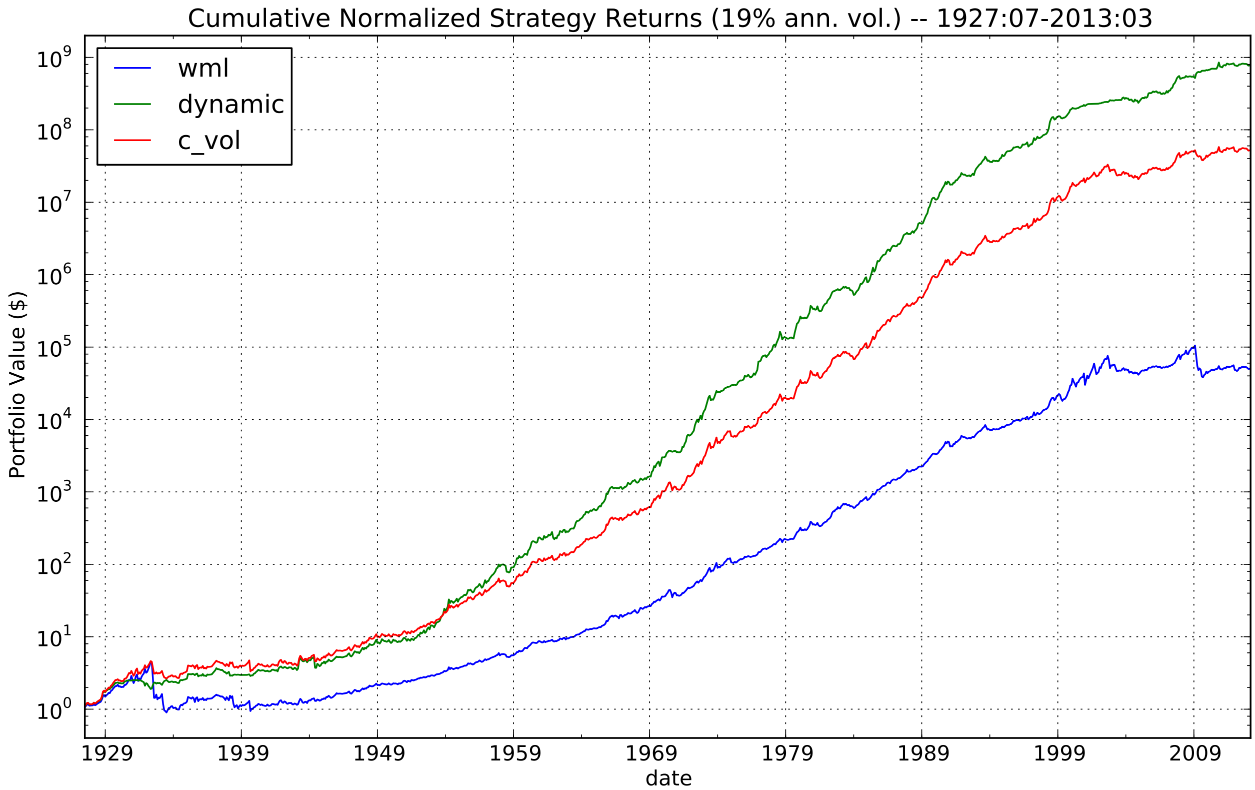

Figure 8: **Dynamic Momentum Strategy Performance**

Plotted are the cumulative returns to the dynamic strategy from equation (4), where λ is chosen so that the in-sample annualized volatility of the strategy is 19%—the same as that of the CRSP value-weighted index over the full sample. For comparison, we also plot the cumulative log returns of the static WML strategy, and a constant volatility strategy similar to that of Barroso and Santa-Clara (2012).

dynamic portfolio varies over time, and indeed is lowest when WML's volatility is forecast to be high (and it's mean return low). This is precisely why, in the next subsection, we will show that the performance of the dynamic strategy is higher than the constant volatility strategy.

3.1 Dynamic Strategy Performance

Figure 8 plots the cumulative returns to this dynamic strategy, where λ is chosen so that the in-sample annualized volatility of the strategy is 19%—the same as that of the CRSP value-weighted index over the full sample. For comparison, we also plot the cumulative log returns of the static WML strategy, and a constant volatility strategy similar to that of Barroso and Santa-Clara (2012). As Figure 8 shows, the dynamic portfolio outperforms the constant volatility portfolio, which in turn outperforms the basic WML portfolio.

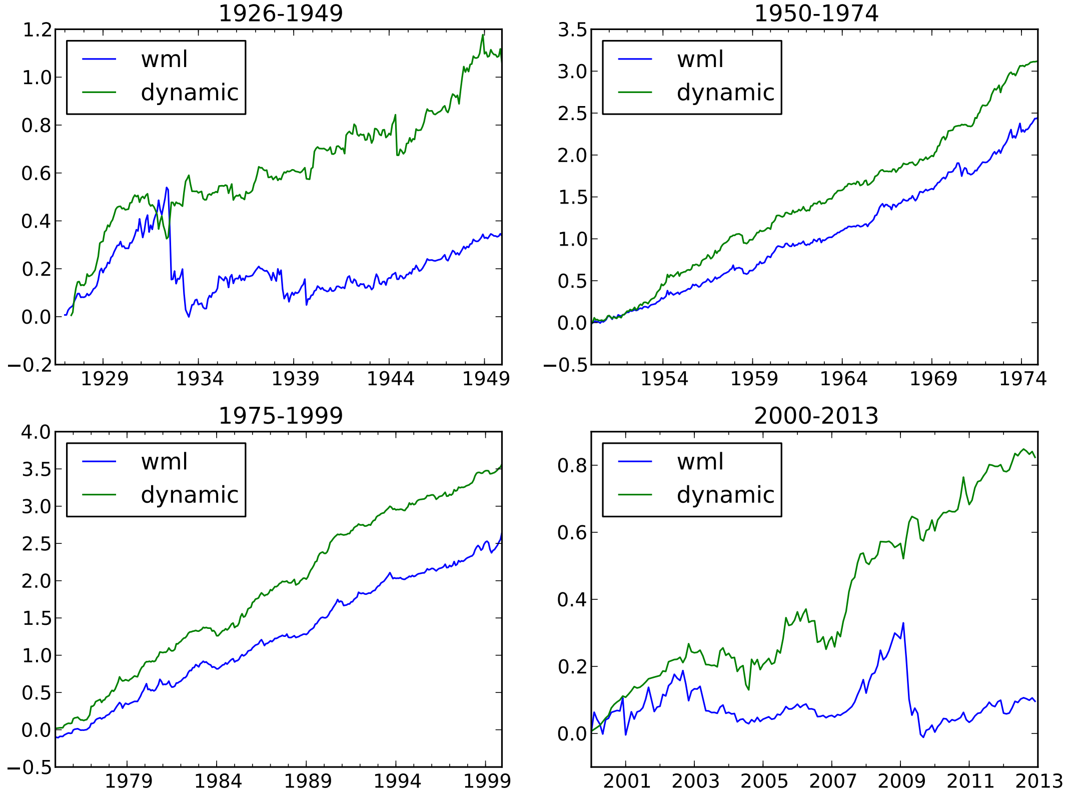

Figure 9: **Cumulative Log Returns – Subperiod Analysis**
Plotted are the cumulative log_{10} returns of the static and dynamic WML strategies, by subsample from: 1927-1950, 1950-1975, 1976-2000, and 2000-2013. Returns for each of the three strategies are scaled so as to make the annualized volatility in each subsample 19%, for ease of comparison.

3.2 Subsample Performance

As a check on the robustness of our results, we perform this same analysis over a set of approximately quarter century subsamples: 1927-1950, 1950-1975, 1976-2000, and 2000-2013. We use the same mean and variance forecasting equation and the same calibration in each of the four subsamples. Table 8 presents the strategy Sharpe ratios and monthly skewness (in parentheses) by subsample, and Figure 9 plots the cumulative log returns by subsample. For this plot, returns for each of the three strategies are scaled so as to make the annualized volatility in each subsample 19%, for ease of comparison.

In each of these subsamples, the ordering of the strategy cumulative performance remains the same: the dynamic strategy outperforms the constant volatility strategy, which outperforms

Table 8: **Dynamic Strategy Performance by Subsample**

This table presents the annualized Sharpe Ratios and skewness (in parentheses) of the three monthly strategies over the full sample (first row), and over four non-overlapping subsamples. The strategy labeled "WML" is our baseline \$1-long/\$1-short WML portfolio. The strategy labeled "const. σ." is a zero-investment strategy for which the long- and short- weights are scaled by the WML strategy's forecast return volatility. The strategy labeled "dynamic" is the dynamic strategy described in Section 3. Here the portfolio weights each month are proportional to the forecast mean return of the WML strategy, dividend by the square of the forecast volatility. Details on the volatility estimation are given in Appendix C.

	Strategy Sharpe Ratio (skewness)		
Subperiod	WML	const. σ	dynamic
1927:01-2013:03	0.60	1.01	1.18
	(-4.70)	(-0.76)	(0.09)
1927:01-1949:12	0.27	0.63	0.68
	(-3.38)	(-1.25)	(-0.99)
1950:01-1974:12	1.31	1.51	1.67
	(-1.16)	(-0.54)	(-0.05)
1975:01-1999:12	1.48	1.69	1.85
	(-0.78)	(-0.41)	(0.18)
2000:01-2013:03	0.27	0.59	0.96
	(-1.50)	(-0.68)	(0.14)

the \$1-long/\$1-short static WML strategy.[11] The skewness numbers in Table 8 also show that the dynamic and constant volatility strategies significantly reduce the negative skewness of returns relative to the constant weight \$1 long-\$1 short strategy in every subsample. Hence, part of the improved performance of the constant volatility and especially dynamic strategy over the static WML portfolio is the amelioration of the big crashes, but even over sub-periods devoid of those crashes, there is still some improvement.

4 Exposure to other risk factors

We have shown that the time varying exposure to market risk cannot explain the low returns of the momentum portfolio in "crash" states. Here, we explore whether the conditional risk associated with either the size and value factors as defined by Fama and French (1993) or exposure to volatility risk can help to explain the conditional returns of the momentum

[11]For comparison, we also show the subsample performance of the constant volatility strategy, where the strategy is scaled to a constant level each period. However, in Figure 8 and Figure 9 the constant volatility strategy is also scaled ex-post to make the realized volatility equal to 19% (annualized).

Table 9: **Regression of WML returns on Variance Swap Returns**

This table presents the results of three daily time-series regressions of the zero-investment WML portfolio returns on an intercept α, on the normalized *ex-ante* forecasting variable $\mathrm{I}_{B,t-1}\hat{\sigma}^2_m$, described in the text, and on this forecasting variable interacted with the excess market return and the return on a (zero-investment) variance swap on the S&P 500 (See Appendix D) The sample is January 2, 1990 to March 28, 2013. The coefficients for α and $\mathrm{I}_{B,t-1}\hat{\sigma}^2_{m,t-1}$ are converted to annualized, percentage terms by multiplying by $252 \cdot 100$.

RHS vars.	(1)	(2)	(3)
α	31.48	29.94	30.39
	(4.7)	(4.8)	(4.9)
$\mathrm{I}_{B\sigma^2}$	-58.62	-49.26	-55.01
	(-5.2)	(-4.8)	(-5.3)
$\tilde{r}^e_{m,t}$		0.109	0.105
		(4.5)	(3.3)
$\mathrm{I}_{B\sigma^2} \cdot \tilde{r}^e_{m,t}$		-0.518	-0.629
		(-28.4)	(-24.7)
$\tilde{r}_{vs,t}$			-0.008
			(-0.2)
$\mathrm{I}_{B\sigma^2} \cdot \tilde{r}_{vs,t}$			-0.101
			(-4.8)

portfolios in crash states.

4.1 Exposure to Volatility Risk

The option-like behavior of the momentum portfolio raises the intriguing question of whether the premium associated with momentum might be related to exposure to variance risk: the results of Section 2.6 show that, in panic states, a long-short momentum portfolio behaves like a short (written) call option on the market. We know, historically, shorting options (*i.e.*, selling variance) has earned a large premium.[12] Hence, we investigate whether the premium earned by momentum can be attributed to it's exposure to variance, and whether this premium changes in panic states.

To assess the dynamic exposure of the momentum strategy to variance, we regress daily WML returns on the daily returns to a variance swap on the S&P 500, which we calculate using the VIX and S&P 500 returns. Appendix D provides details of the calculation of the variance swap return.

As before, we run a time-series regression with a conditioning variable designed to capture

[12]See Christensen and Prabhala (1998) and Carr and Wu (2009).

the time-variation in factor loadings on the market, and potentially on other variables. The conditioning variable is the interaction variable used earlier—$\mathrm{I}_{\mathrm{B}\sigma^2} \equiv (1/\bar{v}_B)\mathrm{I}_{B,t-1}\hat{\sigma}^2_{m,t-1}$—but with a slight twist:

- I_{B} is the bear market indicator defined earlier, where $\mathrm{I}_{\mathrm{B}} = 1$ when the cumulative 2 year market return is negative, through the end of the preceding month, and is zero otherwise.
- $\hat{\sigma}^2_m$ is the variance of the market excess return over the preceding 126 days, as defined earlier.
- $(1/\bar{v}_B)$ is the the inverse of the full-sample mean of $\hat{\sigma}^2_m$ over all months in which $\mathrm{I}_{B,t-1} = 1$.

Normalizing the interaction term with the constant $1/\bar{v}_B$ does not affect the statistical significance of the results. Rather, it gives the coefficients above a simple interpretation. Specifically, given this scaling:

$$\sum_{\mathrm{I}_{B,t-1}=1} \mathrm{I}_{B\sigma^2} = 1.$$

This means that the coefficients on either $\mathrm{I}_{B\sigma^2}$ or on variables interacted with $\mathrm{I}_{B\sigma^2}$ can be interpreted as the weighted average change in the corresponding coefficient when in a bear market, where the weight on each observation is proportional to the ex-ante market variance leading up to that month.

The columns of Table 9 present the results of this analysis. In regression (1) the regression intercept (α) estimates the mean return of the WML portfolio when $\mathrm{I}_{B,t-1} = 0$ as 31.48% per year. However, the coefficient on $\mathrm{I}_{B,t-1} = 1$ shows that the weighted-average decline in "panic" periods (volatile bear markets) is almost 59% per year.

Regression (2) controls for the market return and conditional market risk. Consistent with our earlier results, the final coefficient in this column shows that the estimated WML beta falls by 0.518 ($t = -28.4$) in panic states. However, both the mean WML return in good periods and the change in the WML premium in the panic periods (given, respectively, by α and the coefficient on $\mathrm{I}_{B\sigma^2}$), remain about the same.

In regression (3), we add the return on the variance swap and its interaction with panic states. The coefficient on $\tilde{r}_{vs,t}$ shows that outside of panic states (*i.e.*, when $\mathrm{I}_{B,t-1} = 0$), the

WML return does not covary significantly with the variance swap. However, the coefficient on $\mathrm{I}_{B\sigma^2} \cdot \tilde{r}_{vs,t}$ shows that in panic states, WML has a strongly significant negative loading on the variance swap return. That is, WML is effectively "short volatility" during these periods. This is consistent with the results in Section 2.6, where we showed that WML behaves like a short call option, but only in panic periods; outside of these periods, there is no evidence of any optionality.

Finally, note that the intercept (α) and $\mathrm{I}_{B\sigma^2}$ terms in regression (3), which give the estimated momentum premium in normal and panic states, are essentially unchanged, even after controlling for the variance swap return. The estimated WML premium in non-panic states remains large, and the change in this premium in panic states is just as negative as before, indicating that although momentum returns are related to variance risk, they are not explained by it.

4.2 Exposure to Size and Value Factors

The first two columns of Table 10 present the results of regressing the WML momentum portfolio returns on the three Fama and French (1993) factors consisting of the CRSP VW index return in excess of the risk-free rate, a small minus big (SMB) stock factor, and a high BE/ME minus low BE/ME (HML) factor, all obtained from Ken French's website. In addition, we interact each of the factors with the panic state variable $I_{B\sigma^2}$.

The first two columns show that the abnormal performance for WML continues to be significantly more negative in bear market states, whether we measure abnormal performance relative to the market model or to the Fama and French (1993) three-factor model, with little difference in the point estimates.

The next two columns of the table repeat the market model regressions using HML as the dependent variable instead of WML. For these regressions, we use the modified HML portfolio of Asness and Frazzini (2011). Specifically, Asness and Frazzini show that the Fama and French (1993) HML construction, by using lagged market prices in its BE/ME calculations, inherently induces some positive covariance with momentum. They advocate using the most recent (last month's) price to compute BE/ME ratios in constructing their HML factor, which they term HML-devil (HML-d), in order to examine the value effect separately from momentum. As Table 10 shows, the abnormal return of the HML portfolio increases in the panic states,

Table 10: **Conditional Estimation of WML, HML and SMB premia**

This table presents the results of monthly time-series regressions. The dependent variable is indicated at the head of each column, and is either: (1) *WML*; (2) the *HML–devil* portfolio proposed by Asness and Frazzini (2011); the *SMB* portfolio return of Fama and French (1993); or (4) a portfolio which is 50% *WML* and 50% *HML-devil*. The independent variables are: intercept α, on the normalized *ex-ante* forecasting variable $\mathrm{I}_{B\sigma^2} \equiv (1/\bar{v}_B)\mathrm{I}_{B,t-1} \cdot \hat{\sigma}_m^2$, and on this forecasting variable interacted with the excess market return and the Fama and French (1993) HML and SMB returns. The sample is January 1927 through March, 2013 for the WML and SMB regressions, and January 1927-December 2012 for the HML-d and WML+HML-d portfolios. The coefficients for α and $I_{B,t-1}\hat{\sigma}_m^2$ are converted to annualized, percentage terms by multiplying by $12 \cdot 100$.

	Dependent Variable – Return Series							
Vars.	WML		HML-d		SMB		WML+HML-d	
intercept	24.93	26.95	2.96	3.14	1.91	0.34	13.41	14.08
	(8.6)	(9.4)	(1.8)	(2.2)	(1.5)	(0.3)	(10.0)	(11.0)
$I_{B\sigma^2}$	-28.80	-26.94	9.32	6.57	5.33	5.44	-12.01	-11.13
	(-5.8)	(-5.4)	(3.3)	(2.7)	(2.4)	(2.6)	(-5.2)	(-5.1)
r_m^e	-0.17	-0.15		-0.02		0.21		-0.09
	(-3.3)	(-2.7)		(-0.6)		(9.4)		(-4.0)
$I_{B\sigma^2} \cdot r_m^e$	-0.54	-0.44		0.33		-0.01		-0.11
	(-12.9)	(-7.8)		(16.3)		(-0.4)		(-5.8)
r_{SMB}		-0.16						
		(-1.9)						
$I_{B\sigma^2} \cdot r_{\text{SMB}}$		-0.18						
		(-2.2)						
r_{HML}		-0.38						
		(-4.8)						
$I_{B\sigma^2} \cdot r_{\text{HML}}$		0.05						
		(0.7)						

the opposite of what we find for momentum. This is not surprising for several reasons. First, momentum strategies buy past winners and sell past losers, while value strategies typically buy longer-term past losers and sell winners (see DeBondt and Thaler (1987) and Fama and French (1996)). Also, the correlation between HML-d and UMD is approximately -0.50. Finally, this result is consistent with the intuition for why the market-beta of the WML portfolio changes with past market-returns. Since growth (low book-to-price) stocks have generally had high past returns and value stocks low past returns, the same intuition discussed earlier suggests that HML's beta should be high when $I_{B,t-1} = 1$, and it is. Specifically, HML's market beta is higher by 0.33 when $I_{B,t-1} = 1$ (t-stat = 16.3), as indicated by the interaction term. More directly, the correlation of HML with the excess return on the market during panic states is 0.59, but during normal times it is -0.10. Conversely, for the WML portfolio, the correlation with the market is 0.02 during normal times and -0.71 when $\mathrm{I}_{B,t-1} = 1$.

The next two columns of Table 10 repeat this exercise using SMB as the dependent variable.

The premium on SMB is statistically significantly higher in panic states as well, but its beta does not change significantly during these states. This makes sense since size is a poor proxy for recent short-term performance.

Finally, the last two columns run regressions for a 50-50 equal combination of WML and HML-d following Asness, Moskowitz, and Pedersen (2013), who show that a combination of value and momentum diversifies away a variety of exposures including aggregate market and liquidity risks. Given the opposite-signed results for WML and HML-d on the panic state variables, it is not surprising that a combination of WML and HML-d hedges some of this risk. However, since the magnitude of the effects on WML are much larger than those of HML, the net effect is still a reduction in returns and a decrease in beta during panic states for the momentum-value combination.[13]

5 International Equities and Other Asset Classes

In the academic literature, momentum effects were first documented in individual equities in the United States. Subsequent research has demonstrated the existence of strong momentum effects both among common stocks in other investment regions and in other asset classes.[14]

We investigate the extent to which the same momentum crash patterns we observe in US equities are also present in these other asset markets: first in international equity markets and then in other asset classes.

5.1 Data Construction

5.1.1 International Stock Market Data

The data come from Asness, Moskowitz, and Pedersen (2013). The international stock markets we analyze are the U.S., U.K., Japan, and Continental Europe. We also examine a global

[13] One possibility for the dominance of momentum here is that the 50-50 momentum-value weighting is based on equal dollar allocation to both rather than equal *risk* allocation. Since momentum is more volatile than value, this may be tilting the overall exposure of the combination portfolio more toward momentum.

[14] The research on this topic is cited in footnote 1 and a summary of these effects is found in Asness, Moskowitz, and Pedersen (2013).

equity momentum strategy *GLB*, which weights each region's equity momentum strategy by the *ex-post* volatility of the portfolio over the full sample.

The source for the U.S. data is CRSP. For the other regions, we use the Datastream and the BARRA International universes. Data description and construction are identical to those in Asness, Moskowitz, and Pedersen (2013), and details on both can be found there. The samples for the U.S. and U.K. begin in January 1972. Continental Europe and Japan begin in February, 1974. For each market, the last month of our sample is May, 2013.[15]

The market portfolios we use for each region are from MSCI: specifically the MSCI US, MSCI UK, MSCI Europe, MSCI Japan indices, and the MSCI World index for the global strategy.

5.1.2 Data for Other Asset Classes

The data, again, come from Asness, Moskowitz, and Pedersen (2013). Specifically, we use equity country index futures across 18 developed equity markets beginning in January 1978, 10 currencies across developed markets starting in January 1979, 10 country government bonds beginning January 1982, and 27 different commodity futures beginning in January 1972. All series end in May 2013.

In addition, we examine two composite portfolios: GA is a global momentum strategy across the non-equity asset classes, which weights each asset class momentum strategy portfolio by the *ex-post* volatility of that portfolio. GAll is a global momentum strategy across all of the equity and non-equity asset classes, which weights the GLB and GA portfolios by their *ex-post* return volatilities over the full sample.

As with our cross-sectional equity strategies, the definition of the market index is different for each asset class: it is the MSCI World index for country index futures, an equal-weighted average of all country bonds for bond markets, an equal-weighted average of all currencies for currency markets, and the Goldman Sachs Commodity Index (GSCI) for commodities.

[15] These data extend beyond the original sample period used in Asness, Moskowitz, and Pedersen (2013), since the data are updated monthly following the same procedure for portfolio construction in Asness, Moskowitz, and Pedersen (2013). The data are available from Toby Moskowitz's website: (http://faculty.chicagobooth.edu/tobias.moskowitz/research/data.html)

5.2 Cross Sectional Equity Momentum Outside the US

The portfolio formation procedure here is similar to that used earlier in this paper. Our momentum measure is each stock's cumulative return from 12 months prior to the formation date to one month prior to the formation date. However in our analysis in Sections 2 and 3, we use the top and bottom decile portfolios, value-weighted. Here, we use the Asness, Moskowitz, and Pedersen (2013) P3−P1 momentum portfolios. This long-short portfolio is less extreme: it is long the one-third of the securities with the highest momentum at the start of each month, and takes short positions in the one-third of the stocks with the lowest momentum. Both the long- and the short-portfolio are value weighted. As documented in Asness, Moskowitz, and Pedersen (2013), over this time period there are strong momentum effects in each of the regions, except Japan. In addition, there is significant co-movement across the strategies, but they are not perfectly correlated.

Panels A through D of Table 11 present the results of the regressions run in Section 2, but here for the other universes. Panel A shows the estimated coefficients and t-statistics from regression equation (1):

$$\tilde{R}_t^{P3-P1} = (\alpha_0 + \alpha_B I_{B,t-1}) + (\beta_0 + \beta_B I_{B,t-1})\tilde{R}^e_{m,t} + \tilde{\epsilon}_t. \tag{7}$$

As noted earlier, in each regression, the excess market portfolio return $\tilde{R}^e_{m,t}$ corresponds to the US dollar return of the stock market within which the momentum strategy is constructed, net of the US treasury-bill rate.

Consistent with the results presented earlier, the market betas of the momentum strategy are dramatically lower in bear markets across the other stock markets as well. The strategies implemented using European and Japanese stocks have market betas that are approximately 0.5 lower during bear markets (with t-stats of about -7). The UK momentum strategy beta falls by 0.21. The drop in this period for this US momentum strategy is 0.58 – comparable to what we observe for the WML portfolio over the longer 1927 to 2013 period. Globally, averaging across the US, UK, Europe, and Japan, the market betas of the momentum strategy are markedly lower in bear markets.

The abnormal return of the momentum strategies is significantly positive in bull markets for all regions except Japan. Consistent with our analysis in Section 2, the abnormal return is lower in bear markets in each region, though using only the bear market indicator as a proxy for panic periods none of the differences are statistically significant over this shorter sample period.

Panel B investigates the optionality in the momentum strategy in bear markets using regression equation (2)

$$\tilde{R}_t^{P3-P1} = (\alpha_0 + \alpha_B I_{B,t-1}) + (\beta_0 + I_{B,t-1}[\beta_B + \tilde{I}_{U,t}\beta_{B,U}])\tilde{R}_{m,t}^e + \tilde{\epsilon}_t. \tag{8}$$

Consistent with the longer period US results, there is statistically significant optionality in bear markets in the EU, UK, and JP stock markets, and globally across all markets. Interestingly, for this subsample and methodology, the optionality is of the right sign, but is not statistically significant for the US market. The negative beta of long-short momentum strategies is particularly acute when the contemporaneous market return is positive. That is, momentum strategies in all regions across the world exhibit conditional betas and payoffs similar to writing call options on the local market.

In Panel C, we add as a conditioning variable the realized daily market return variance, annualized, over the preceding 126 trading days (6 months).[16]

$$\tilde{R}_t^{P3-P1} = [\alpha_0 + \alpha_B I_{B,t-1} + \alpha_V \hat{\sigma}_{m,t-1}^2] + [\beta_0 + \beta_B I_{B,t-1} + \beta_V \hat{\sigma}_{m,t-1}^2]\tilde{R}_{m,t}^e + \tilde{\epsilon}_t. \tag{9}$$

Two interesting results emerge. First, higher *ex-ante* market variance is generally associated with more negative momentum strategy betas. Second, higher market variance is also associated with strongly lower future abnormal returns to momentum, net of the market return. This last relation is statistically significant in all markets, and again is consistent with our earlier results for the US market over the longer period.

In Panel D we again use the $\mathrm{I}_{B\sigma^2} \equiv (1/\bar{v}_B)\mathrm{I}_{B,t-1} \cdot \hat{\sigma}_m^2$ measure introduced in Section 4, designed to capture "panic" periods when the the market has fallen and volatility is high. In addition, in these regressions we instrument for time variation in market beta using $I_{B,t-1}$, $\hat{\sigma}_{m,t-1}^2$, and

[16] This is the same market variance measure used earlier in the paper. However, for the EU, JP, and UK regions we have daily MSCI market return data only for the time period from January 1990 on. Therefore, over the period from 1972:01-1990:06 in the UK, and 1974:01-1990:06 in the EU and JP, we use the realized monthly variance over the preceding 6 months, again annualized.

$I_{B\sigma^2}$. Specifically, we run the regression

$$\tilde{R}_t^{P3-P1} = [\alpha_0 + \alpha_B I_{B\sigma^2}] + [\beta_B I_{B,t-1} + \beta_V \hat{\sigma}^2_{m,t-1} + \beta_{I_B V} I_{B\sigma^2}]\tilde{R}^e_{m,t} + \tilde{\epsilon}_t. \tag{10}$$

The results in Panel D of Table 11 are remarkably consistent with our earlier results for the US over the longer period. The coefficient on the interaction $I_{B\sigma^2}$ term is negative, economically large, and statistically significant in all markets, and for the global strategy. In summary, the results in this table suggest that momentum strategies in these different equity markets are also short volatility, and have significantly lower abnormal returns in panic periods characterized by poor lagged market returns and high market volatility.

One other point of interest is that, in Panels C and D of Table 11, the $\hat{\alpha}$ for the Japan momentum strategy is considerably larger, and in Panel C is in fact significant at a 5% level. We'll explore the implications of this further in Section 5.4, where we will explore a dynamic Japanese momentum strategy which takes into account both the forecastability in the expected return and the forecastability of the strategy volatilty.

5.3 Momentum in Other Asset Classes

The previous subsection showed that the option-like payoffs of momentum strategies in bear markets is a feature present outside of US equity markets and indeed in every other equity market we examined. These findings give credence to this feature of momentum being a robust phenomenon and not likely due to chance. In addition, because these other markets are all equity momentum strategies, the results may be consistent with Merton (1974). Common stocks that have lost significant value, particularly in bear markets, are like out of the money call options on the firm, and consequently should exhibit option-like behavior.

For further robustness on the optionality of momentum and to further test the Merton (1974)-type theory for its existence, we examine momentum strategies on the non-equity asset classes that include government bonds, currencies, and commodity futures. These other asset classes provide another out of sample test for the option-like payoffs of momentum strategies in bear markets. However, it is unlikely that a Merton (1974)-type story would explain such a feature in these asset classes. Hence, finding the same option-like asymmetry in these asset classes will provide additional robustness, but would also present a challenge to the Merton (1974)

Table 11: **Time Series Regressions for International Equity Markets**

This table below reports the estimated coefficients and t-statistics from regressions of the monthly returns to zero-investment equity momentum strategy in each region on the indicated set of RHS variables. The estimated intercept, and the coefficients on $I_{B,t-1}$ and $\mathrm{I}_{B\sigma^2}$ are all multiplied by 12·100 to put them in annualized, percentage terms. GLB is global equity momentum strategy described in Section 5.1.1.

	EU	JP	UK	US	GLB
start	1974-02	1974-02	1972-01	1972-02	1972-01
end	2013-05	2013-05	2013-05	2013-05	2013-05
			Panel A		
α	8.935	1.887	7.409	5.181	5.826
	(3.5)	(0.5)	(2.7)	(1.9)	(3.6)
I_B	-3.549	-0.837	-6.827	-2.921	-4.920
	(-0.7)	(-0.1)	(-1.1)	(-0.5)	(-1.2)
R_m^e	0.071	0.246	0.015	0.150	0.023
	(1.6)	(4.8)	(0.4)	(2.7)	(0.7)
$I_B R_m^e$	-0.508	-0.527	-0.197	-0.584	-0.275
	(-7.1)	(-7.0)	(-3.1)	(-6.2)	(-4.6)
			Panel B		
α	8.935	1.887	7.409	5.181	5.826
	(3.6)	(0.5)	(2.7)	(1.9)	(3.6)
I_B	9.418	11.104	4.249	-0.266	5.019
	(1.2)	(1.3)	(0.5)	(-0.0)	(0.8)
R_m^e	0.071	0.246	0.015	0.150	0.023
	(1.7)	(4.8)	(0.4)	(2.7)	(0.7)
$I_B R_m^e$	-0.302	-0.318	0.004	-0.540	-0.098
	(-2.7)	(-2.5)	(0.0)	(-3.3)	(-1.0)
$I_B I_U R_m^e$	-0.418	-0.367	-0.306	-0.086	-0.342
	(-2.4)	(-2.0)	(-2.2)	(-0.3)	(-2.2)
			Panel C		
α	12.237	12.385	10.856	10.331	8.345
	(4.1)	(2.5)	(3.6)	(3.4)	(4.8)
I_B	1.445	4.554	0.213	6.018	2.254
	(0.3)	(0.7)	(0.0)	(0.9)	(0.5)
$\hat{\sigma}_m^2$	-0.113	-0.221	-0.078	-0.204	-0.252
	(-2.0)	(-2.9)	(-2.6)	(-3.3)	(-3.7)
R_m^e	0.115	0.280	0.020	0.215	0.041
	(2.5)	(4.2)	(0.5)	(3.6)	(1.2)
$I_B R_m^e$	-0.391	-0.512	-0.182	-0.485	-0.206
	(-4.8)	(-6.5)	(-2.5)	(-4.8)	(-3.2)
$\hat{\sigma}_m^2 R_m^e$	-1.755	-0.734	-0.040	-2.361	-1.959
	(-2.6)	(-0.7)	(-0.2)	(-2.5)	(-2.2)
			Panel D		
α	10.286	5.333	8.627	7.084	6.720
	(4.4)	(1.6)	(3.4)	(2.8)	(4.5)
$I_{B\sigma^2}$	-6.509	-9.910	-11.408	-11.055	-8.704
	(-2.0)	(-2.2)	(-3.2)	(-2.6)	(-3.6)
$I_B R_m^e$	-0.306	-0.180	-0.176	-0.245	-0.177
	(-3.7)	(-1.8)	(-2.6)	(-2.4)	(-2.8)
$\hat{\sigma}_m^2 R_m^e$	-0.295	3.685	-0.600	1.839	-2.798
	(-0.2)	(3.8)	(-0.8)	(1.2)	(-1.2)
$I_{B\sigma^2} R_m^e$	-0.056	-0.307	0.073	-0.261	0.036
	(-0.7)	(-3.2)	(0.8)	(-2.4)	(0.5)

explanation.

We use the zero-investment P3−P1 momentum portfolios in each non-equity asset class from Asness, Moskowitz, and Pedersen (2013), which are the top third minus bottom third of assets within each asset class based on their prior 12-month cumulative return, skipping the most recent month, and where securities are equal-weighted within these asset classes (since there is no notion of size or market cap for futures or currencies). From Asness, Moskowitz, and Pedersen (2013) we know that every momentum strategy (except the bond strategy) produces returns that are different from zero at conventional significance levels. We also examine the "all" strategy from Asness, Moskowitz, and Pedersen (2013), which is an equal volatility weighted average portfolio across the four asset classes and an "all+stock" portfolio which combines the four asset-class and the four international equity momentum strategies.

Table 12 presents the results of time series regressions for the asset-class momentum strategies similar to those in Table 11 for the international equity momentum strategies. Panels A, B, C, and D report the results of estimating equations (7), (8), (9), and (10), respectively.

The patterns revealed in Table 12 are similar to what we see in international equities. First, the set of $\mathrm{I}_{B,t-1} \cdot \tilde{R}^e_m$ coefficients and t-statistics in the last row of Panel A show that, in all asset classes, the momentum portfolio's market beta is significantly more negative in bear markets. The intuition that, following a bear market, the loser side of the momentum portfolio will have a high market beta is valid for other asset classes as well.

The $\mathrm{I}_{B,t-1}$ coefficients in the second row of Panel A provide evidence weakly consistent with the earlier finding that market-risk adjusted momentum returns are lower following bear markets. The point estimates are all negative, except for bonds, but only in the currency market is the $I_{B,t-1}$ coefficient statistically significant.

The set of regressions in Panel B help to assess whether the optionality present in cross-sectional equity momentum strategies is also present here in other asset classes. The $\mathrm{I}_{B,t-1}\tilde{\mathrm{I}}_{U,t}\tilde{R}^e_{m,t}$ coefficient is negative for each of the four asset classes, and the two composite portfolios, but is statistically significant at a 5% level only for commodities. This result is intriguing. While a model such as Merton (1974) would argue that equities would exhibit option-like features, it is not clear that such a model would easily explain the optionality present in currency and commodity futures markets.

Table 12: **Time Series Regressions for other Asset Classes**

This table below reports the estimated coefficients and t-statistics from regressions of the monthly returns to zero-investment momentum strategies in each asset class on the indicated set of RHS variables. *GA* and *GAll* are the global strategies described in Section 5.1.2. The estimated intercept, and the coefficients on $I_{B,t-1}$ and $\mathrm{I}_{B\sigma^2}$ are all multiplied by 12·100 to put them in annualized, percentage terms.

	FI	CM	FX	EQ	GA	GAll
start	1983-02	1973-02	1980-02	1979-02	1973-02	1973-02
end	2013-05	2013-05	2013-05	2013-05	2013-05	2013-05
			Panel A			
α	0.006	16.302	4.745	8.575	4.653	4.639
	(0.0)	(3.7)	(2.2)	(3.8)	(4.6)	(5.0)
I_B	0.798	-10.470	-8.221	-0.575	-2.426	-3.294
	(0.3)	(-1.4)	(-2.5)	(-0.1)	(-1.1)	(-1.3)
R_m^e	0.186	0.308	0.382	0.272	0.162	0.082
	(3.1)	(4.1)	(4.4)	(6.0)	(2.7)	(1.9)
$I_B R_m^e$	-0.362	-0.730	-1.092	-0.620	-0.485	-0.366
	(-2.7)	(-4.5)	(-8.6)	(-8.5)	(-3.9)	(-4.4)
			Panel B			
α	0.006	16.302	4.745	8.575	4.653	4.639
	(0.0)	(3.8)	(2.2)	(3.8)	(4.6)	(5.1)
I_B	1.994	7.014	-4.096	6.248	1.142	3.746
	(0.6)	(0.7)	(-0.9)	(1.1)	(0.4)	(1.1)
R_m^e	0.186	0.308	0.382	0.272	0.162	0.082
	(3.1)	(4.1)	(4.4)	(6.0)	(2.7)	(1.9)
$I_B R_m^e$	-0.278	-0.205	-0.911	-0.485	-0.222	-0.106
	(-1.4)	(-0.8)	(-5.0)	(-4.6)	(-1.2)	(-0.9)
$I_B I_U R_m^e$	-0.197	-1.102	-0.405	-0.312	-0.563	-0.605
	(-0.6)	(-2.5)	(-1.4)	(-1.7)	(-1.8)	(-2.9)
			Panel C			
α	-0.297	20.050	7.527	9.277	5.835	5.963
	(-0.2)	(3.5)	(2.4)	(3.8)	(4.9)	(5.9)
I_B	1.057	-9.022	-7.475	0.634	-0.759	0.554
	(0.4)	(-1.2)	(-2.2)	(0.2)	(-0.3)	(0.2)
$\hat{\sigma}_m^2$	0.136	-0.211	-0.503	-0.047	-0.756	-0.585
	(0.2)	(-1.1)	(-1.2)	(-0.7)	(-1.8)	(-3.0)
R_m^e	0.278	0.522	0.429	0.299	0.201	0.104
	(2.1)	(4.3)	(4.0)	(6.3)	(3.0)	(2.3)
$I_B R_m^e$	-0.385	-0.712	-1.045	-0.549	-0.374	-0.267
	(-2.8)	(-4.4)	(-8.0)	(-6.6)	(-2.7)	(-2.8)
$\hat{\sigma}_m^2 R_m^e$	-55.971	-8.820	-9.702	-2.001	-23.842	-8.605
	(-0.8)	(-2.2)	(-0.8)	(-1.7)	(-1.2)	(-1.5)
			Panel D			
α	0.218	13.803	3.419	9.240	4.766	4.853
	(0.2)	(3.7)	(1.8)	(4.7)	(5.1)	(5.6)
$I_{B\sigma^2}$	0.026	-4.808	-4.655	-2.683	-2.308	-4.056
	(0.0)	(-1.2)	(-2.1)	(-1.2)	(-1.8)	(-2.8)
R_m^e	0.263	0.772	0.672	0.384	0.238	0.128
	(1.9)	(5.0)	(3.0)	(5.9)	(2.0)	(2.0)
$I_B R_m^e$	-0.281	-1.207	-1.293	-0.669	-0.424	-0.303
	(-0.8)	(-4.8)	(-5.0)	(-6.4)	(-2.3)	(-2.7)
$\hat{\sigma}_m^2 R_m^e$	-46.141	-18.887	-60.175	-8.332	-49.075	-22.030
	(-0.6)	(-3.4)	(-1.4)	(-2.4)	(-0.7)	(-1.0)
$I_{B\sigma^2} R_m^e$	-0.105	0.344	0.268	0.222	0.074	0.095
	(-0.3)	(2.5)	(1.3)	(1.9)	(0.4)	(0.6)

Panel C of Table 12 estimates equation (9) for the other-asset-class momentum strategies. Here the signs in the relation between lagged volatility and momentum strategy returns are again negative in the commodity (CM), currency (FX), and equity (EQ) futures asset classes. Panel D again uses the interactive variable $I_{B\sigma^2}$ as an instrument for volatile bear markets. As in Table 11, we control for variation in market beta associated with $I_{B,t-1}$, $\hat{\sigma}_m^2$, and the interaction term itself. In all asset classes except FI, the coefficient on this interaction term is negative, consistent with our previous findings in US and international equity markets. However, except for FX and the GAll portfolio, the coefficient is not significant at a 5% level of significance.

These results are largely consistent with those found for US equities and for all other international equities, too. However, these findings are more difficult to reconcile under a Merton (1974)-style theory, which is better suited for equity returns.

5.4 Robustness of Dynamic Strategies in Other Markets and Asset Classes

Given the robustness of the option-like features to momentum in other equity markets and other asset classes, we examine the efficacy of the dynamic momentum strategy we employed in US equity markets internationally and in other asset classes to see how robust its outperformance is out of sample.

As in our analysis of the US-equity dynamic momentum strategy, the dynamic strategy employed in each alternative asset class is rebalanced monthly. Specifically, at the beginning of each month we buy the value-weighted "winner" portfolio and short the value-weighted "loser"portfolio. However, while for the static WML strategy we buy and short $1 of each portfolio, here we buy or sell $\$w_t$, where w_t is based on the *ex-ante* expected return and volatility of the WML portfolio (as in Appendix B) using the instruments from the previous analysis to forecast the expected return and volatility.

Our forecast of the mean return for the WML portfolio in each asset class is based on a univariate forecast, using the interaction of the ex-ante bear market indicator for that asset class, $I_{B,t-1}$, and the asset-class market volatility over the preceding 6-months.

As in our analysis of the US-equity dynamic momentum strategy, we forecast the WML volatility by first fitting the GARCH-M process proposed by Glosten, Jagannathan, and Runkle (1993) to the WML returns for that asset class. We combine this with a lagged 126 day (*i.e.,* 6 month) measure of WML volatility. Precise specifications of the forecasting model, and the GARCH model parameters for each asset class are given in Appendix C.

Table 13 reports the Sharpe ratio and skewness (in parentheses) of the simple $1 long-$1 short WML momentum strategy in each market and asset class, as well as a constant volatility momentum strategy and a dynamic momentum strategy as constructed in Section 3 for each market and asset class. In addition, we report global combinations of the equity momentum strategies across all markets (GLB), the non-equity asset classes (GA), and a combination of all equity markets and non-equity asset classes (GAll).

As Table 13 shows, there is a marked improvement in Sharpe ratio going from the static WML momentum strategy to a constant volatility momentum strategy to our dynamic momentum strategy in every single market and asset class we study. In most cases, our dynamic strategy doubles the Sharpe ratio over the traditional static momentum portfolio. Furthermore, our dynamic momentum strategy resurrects positive returns in markets where the typical momentum portfolio has failed to produce positive profits, such as Japan. In Japan, the static, classic momentum portfolio delivers a 0.07 Sharpe ratio, but our dynamic momentum portfolio in Japan produces a 0.37 Sharpe ratio. (Alas, even the dynamic strategy does not deliver positive average returns for fixed income.)

The skewness numbers (in parentheses) are also interesting as the predominantly negative skewness of the static momentum strategies across all markets is apparent, but the dynamic momentum strategy delivers mostly positive skewness, which is consistent with the improvement in Sharpe ratio we see from the dynamic strategy.

Finally, the last row of Table 13 reports results for a "fully dynamic" portfolio that is a weighted combination of the individual asset class or market dynamic strategies, where the weighs are based on the *ex-ante* conditional volatility of each component strategy. That is, each of the component strategies is scaled to have equal volatility (ex ante), and then the strategies are equally weighted. In this way, we are also using cross-sectional information on the strength of the dynamic signal of each component strategy to build a fully dynamic combination portfolio across all asset classes. As Table 13 indicates, there is additional Sharpe ratio

Table 13: **Sharpe Ratios of the WML, Dynamic and Constant Volatility strategies**

This table presents the annualized Sharpe ratios of momentum strategies each of the different asset class strategies. For each asset class, WML denotes the baseline \$1-long/\$1-short momentum strategy. "cst.-σ" denotes the strategy which is weighted by the *ex-ante* forecast volatility of the strategy. "dynam." is the maximum sharpe ratio strategy described in appendix B. The final row of the table, labeled "full-dyn" is the Sharpe-ratio of a portfolio which is a weighted combination of the dynamic strategies that make up that "generalized" strategy, where the weighs are based on the *ex-ante* conditional volatility of the each component strategy. That is, each of the component strategies is scaled to have equal volatility, and then the strategies are equally weighted.

	Annualized Strategy Sharpe Ratio (Skewness)										
	EU	JP	UK	US	GE	FI	CM	FX	EQ	GA	GAll
start	06/90	06/90	06/90	07/72	07/72	06/83	02/73	02/80	02/79	02/73	02/73
end	05/13	05/13	05/13	05/13	05/13	05/13	05/13	05/13	05/13	05/13	05/13
Static WML	0.462	0.067	0.465	0.283	0.513	0.004	0.587	0.296	0.705	0.676	0.754
	(-0.34)	(0.02)	(-0.62)	(-0.04)	(-0.34)	(-0.24)	(0.01)	(-0.54)	(-0.18)	(-0.48)	(-0.33)
Const. σ WML	0.823	0.159	0.737	0.519	0.732	0.016	0.685	0.424	0.797	0.790	0.940
	(0.44)	(-0.11)	(-0.04)	(-0.09)	(0.12)	(-0.47)	(-0.07)	(-0.47)	(0.05)	(-0.31)	(-0.18)
Dynamic WML	0.931	0.333	0.826	0.658	0.678	-0.065	0.787	0.624	0.825	0.936	1.114
	(0.82)	(2.08)	(0.50)	(0.12)	(1.22)	(0.35)	(0.30)	(-0.15)	(0.30)	(-0.11)	(0.09)
Full-dynamic WML					0.930					0.980	1.162
					(0.60)					(-0.20)	(0.08)

improvement from this additional twist on our dynamic momentum strategies, providing another robustness test on the use of conditional mean and variance forecastability in enhancing the returns to momentum strategies. Overall, the very consistent evidence of the optionality of momentum strategies, conditional betas and return premia, and improvement from our dynamic weighting scheme across different markets and vastly different asset classes, provides a wealth of out of sample evidence for our findings and suggests that momentum crashes and the ability to forecast these episodes is a reliable feature of momentum-type strategies.[17]

[17] Although beyond the scope of this paper, it would be interesting to see if other momentum-type strategies, such as earnings momentum in equities (Chan, Jegadeesh, and Lakonishok (1996)), or time-series momentum in futures contracts (Moskowitz, Ooi, and Pedersen (2012)), or cross-momentum effects (Cohen and Frazzini (2008)) exhibit similar features.

6 Conclusions

In "normal" environments we see consistent price momentum that is both statistically and economically strong and manifests itself across numerous equity markets and a wide range of diverse asset classes.

However, in extreme market environments following a long market downturn, the market prices of past losers embody a very high premium. When poor market conditions ameliorate and the market starts to rebound, the losers experience strong gains, resulting in a "momentum crash" as momentum strategies short these assets. We find that, in bear market states, and in particular when market volatility is high, the down-market betas of the past-losers are low, but the up-market betas are very large. This optionality does not appear to generally be reflected in the prices of the past losers. Consequently, the expected returns of the past losers are very high, and the momentum effect is reversed during these times. This feature does not apply equally to winners during good times, however, resulting in an asymmetry in the winner and loser exposure to market returns during extreme times.

These results are shown to be remarkably robust across eight different markets and asset classes, as well as multiple time periods. Moreover, since part of these crash periods are predictable, we use bear market indicators and volatility estimates to forecast the conditional mean and variance of momentum strategies. Armed with these estimates, we create a simple dynamically-weighted version of the momentum portfolio that approximately doubles the Sharpe ratio of the static momentum strategy—and does so consistently in every market, asset class, and time period we study.

What can explain these findings? We examine a variety of explanations ranging from compensation for crash risk to volatility risk, to other factor risks such as the Fama and French (1993) factors, but find that none of these explanations can account fully for our findings, though volatility risk goes in the right direction. For equity momentum, a Merton (1974) story for the option-like payoffs of equities may make sense. However, the existence of the same phenomena and option-like features for momentum strategies in futures, bonds, currencies, and commodities, makes this story more challenging. Alternatively, these effects may be loosely consistent with several behavioral findings, where in extreme situations individuals tend to be fearful and appear to focus on losses, largely ignoring probabilities.[18] Whether this behavioral

[18] See Sunstein and Zeckhauser (2008), Loewenstein, Weber, Hsee, and Welch (2001), and Loewenstein

phenomenon is fully consistent with the empirical results documented here is a subject for further research and would indicate that the behavior of market participants in each of these markets and asset classes is affected similarly, despite the fact that the average and marginal investor in these various markets are quite different along many other dimensions.

(2000).

Appendices

A Detailed Description of Calculations

A.1 Cumulative Return Calculations

The cumulative return, on an (implementable) strategy is an investment at time 0, which is fully reinvested at each point – *i.e.*, where no cash is put in or taken out. That is, the cumulative arithmetic returns between times t and T is denoted $R(t, T)$.

$$R(t, T) = \prod_{s=t+1}^{T} (1 + R_s) - 1,$$

where R_s denotes the arithmetic return in the period ending at time t, and $r_s = log(1 + R_s)$ denotes the log-return over period s,

$$r(t, T) = \sum_{s=t+1}^{T} r_s.$$

For long-short portfolios, the cumulative return is:

$$R(t, T) = \prod_{s=t+1}^{T} (1 + R_{L,s} - R_{S,s} + R_{f,t}) - 1,$$

where the terms $R_{L,s}$, $R_{S,s}$, and $R_{f,s}$ are, respectively, the return on the long side of the portfolio, the short side of the portfolio, and the risk-free rate. Thus, the strategy reflects the cumulative return, with an initial investment of V_t, which is managed in the following way:

1. Using the $\$V_0$ as margin, you purchase $\$V_0$ of the long side of the portfolio, and short $\$V_0$ worth of the short side of the portfolio. Note that this is consistent with Reg-T requirements. Over each period s, the margin posted earns interest at rate $R_{f,s}$.
2. At then end of each period, the value of the investments on the long and the short side of the portfolio are adjusted to reflect gains to both the long and short side of the portfolio. So, for example, at the end of the first period, the investments in both the long and short side of the portfolio are adjusted to set their value equal to the total value of the portfolio to $V_{t+1} = V_t \cdot (1 + R_L - R_S + R_f)$.

Note that, for monthly returns, this methodology assumes that there are no margin calls, etc., except at the end of each month. These calculated returns do not incorporate transaction costs.

B Maximum Sharpe Ratio Strategy

The setting is discrete time with T periods from $1, \ldots, T$. We can trade in two assets, a risky asset and a risk free asset. Our objective is to maximize the Sharpe ratio of a portfolio where, each period, we can trade in or out of the risky asset with no cost.

Over period $t+1$ – which is the span from t to $t+1$ – the excess return on a risky asset $\tilde{r}_{t+1}$ is distributed normally, with time-t conditional mean μ_t and conditional variance σ_t^2. That is,

$$\mu_t = \mathbb{E}_t\left[\tilde{r}_{t+1}\right] \quad \text{and} \quad \sigma_t^2 = \mathbb{E}_t\left[(\tilde{r}_{t+1} - \mu_t)^2\right]. \tag{11}$$

Suppose further that at $t = 0$ the agent knows μ_t and σ_t for $t \in \{0, \ldots, T-1\}$.

The agent's objective is to maximize the full period Sharpe ratio of a managed portfolio. The agent manages the portfolio by placing, at the beginning of each period, a fraction w_t of the value of the managed portfolio in the risky asset, and a fraction $1 - w_t$ in the risk-free asset. The time t expected excess return and variance of the managed portfolio in period $t+1$ is then given by:

$$\tilde{r}_{p,t+1} = w_t \tilde{r}_{t+1} \sim \mathcal{N}\left(w_t \mu_t, w_t^2 \sigma_t^2\right).$$

The Sharpe ratio over the T periods is:

$$\text{SR} = \frac{\mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} \tilde{r}_{p,t}\right]}{\sqrt{\mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T}(\tilde{r}_{p,t} - \bar{r}_p)^2\right]}}$$

where the $\bar{r}_p$ in the denominator is the sample average per period excess return ($\frac{1}{T}\sum_{t=1}^{T} \tilde{r}_{p,t}$).

Given the information structure of this optimization problem, maximizing the Sharpe ratio is equivalent to solving the constrained maximization problem:

$$\max_{w_0,\ldots,w_{T-1}} \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} \tilde{r}_{p,t}\right] \quad \textit{subject to} \quad \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T}(\tilde{r}_{p,t} - \bar{r})^2\right] = \sigma_p^2.$$

If the period length is sufficiently short, then $\mathbb{E}[(\tilde{r}_{p,t} - \bar{r})^2] \approx \sigma_t^2 = \mathbb{E}_t\left[(\tilde{r}_{t+1} - \mu_t)^2\right]$. With this approximation, substituting in the conditional expectations for the managed portfolio from equation (11) gives the Lagrangian:

$$\max_{w_0,\ldots,w_{T-1}} \mathscr{L} \equiv \max_{w_t} \left(\frac{1}{T}\sum_{t=0}^{T-1} w_t \mu_t\right) - \lambda\left(\frac{1}{T}\sum_{t=0}^{T-1} w_t^2 \sigma_t^2 = \sigma_p^2\right).$$

The T first order conditions for optimality are:

$$\left.\frac{\partial \mathscr{L}}{\partial w_t}\right|_{w_t=w_t^*} = \frac{1}{T}\left(\mu_t - 2\lambda w_t^* \sigma_t^2\right) = 0 \qquad \forall t \in \{0, \ldots, T-1\}$$

giving an optimal weight on the risky asset at time t of:

$$w_t^* = \left(\frac{1}{2\lambda}\right)\frac{\mu_t}{\sigma_t^2}.$$

That is, the weight placed on the risky asset at time t should be proportional to the expected excess return over the next period, and inversely proportional to the conditional variance.

C GJR-GARCH Forecasts of Momentum Portfolio Volatility

The construction of the dynamic portfolio strategy we explore in Sections 3 and 5.4 requires estimates of the conditional mean return and the conditional volatility of the momentum strategies. To forecast the volatility we first fit a GARCH process to the daily momentum returns of each asset class. Specifically, we fit the GARCH model proposed by Glosten, Jagannathan, and Runkle (1993), and summarized by equations (5) and (6). The maximum likelihood estimates and t-statistics are:

parameter:	$\hat{\mu}$	$\hat{\omega}$	$\hat{\alpha}$	$\hat{\gamma}$	$\hat{\beta}$
ML-est	0.86×10^{-3}	1.17×10^{-6}	0.111	-0.016	0.896
t-stat	(14.7)	(4.2)	(14.4)	(-1.6)	(85.1)

We then regress the future realized 22-day WML return volatility $\hat{\sigma}_{22,t+1}$ on the GJR-GARCH estimate ($\hat{\sigma}_{\text{GARCH,t}}$) and the lagged 126-day WML return volatility ($\hat{\sigma}_{126,t}$), and a constant. The OLS coefficient estimates and t-statistics are:

coefficient:	$\hat{\alpha}$	$\hat{\sigma}_{\text{GARCH,t}}$	$\hat{\sigma}_{126,t}$
coef. est.	0.0010	0.6114	0.2640
t-stat	(3.0)	(16.7)	(7.2)

with a regression $R^2_{\text{adj}} = 0.617$.[19] The fitted estimate of $\hat{\sigma}_{22,t+1}$ is then used as an input to the dynamic WML portfolio weight, as discussed in Sections 3 and 5.4.

[19] The lag one residual autocorrelation is 0.013 (t-stat = 0.44), justifying the use of OLS standard errors. Also, the t-statistics on the lag 2-5 autocorrelations never exceed 1.14. It is interesting that the autocorrelation of the dependent variable of the regression ($\hat{\sigma}_{22,t}$) is large and statistically significant ($\hat{\rho}_1 = 0.55$, t-stat = 24.5). This suggests that the autocorrelation in $\hat{\sigma}_{22,t}$ results from its forecastable component. The residual from its projection on the forecast variables is uncorrelated at any conventional statistically significant level.

The same estimation procedure is used to generate a forecast of the future 22-day WML return volatility in each of the alternative asset classes. The maximum-likelihood GJR-GARCH parameter estimates and t-statistics, and regression estimates and t-statistics are presented in Table 14.

The parameters above and in Table 14 tell an interesting story: first, in the regressions, the coefficient on the GJR-GARCH estimate of volatility is always significant, and the coefficient on the lagged 126-day volatility is always smaller, but not always statistically significant. There appears to be a longer-lived component of volatility that $\hat{\sigma}_{126,t}$ is capturing.

Also interesting is the leverage parameter γ. In each of the asset classes, the maximum-likelihood estimate of γ is negative: this means that a strong negative return on the WML portfolio is generally associated with a *decrease* in the WML return variance. As noted elsewhere in the literature, this coefficient is positive at high levels of statistical significance for the market return (see, *e.g.*, Glosten, Jagannathan, and Runkle (1993) and Engle and Ng (1993).)

Table 14: **Maximum Likelihood Estimates of GJR-GARCH Model for Momentum Portfolios**

The upper panel of this table presents the maximum likelihood estimates of the coefficients of the GJR-GARCH model – given by equations (5) and (6) – fitted to each of the momentum portfolios we examine in Section 5.4. Start and end dates for each of the 11 daily return series are also given in yyyy-mm-dd format. Note that the estimates of the μ and ω coefficients are multiplied by 10^3 and 10^6, respectively. Maximum Likelihood based t-statistics are given in parentheses. For β, this t-statistic tests whether $\beta = 1$; for all other parameters it tests whether the parameter is zero. The lower panel presents the results of the monthly regressions in which we regress the future one month daily volatility of the WML portfolio on an intercept (α), on the lagged 126 day WML return volatility (σ_{126}), and on the lagged GJM-GARCH volatility (σ_{GARCH}).

	GARCH estimates by Asset Class										
	EU	JP	UK	US	GLB	FI	CM	FX	EQ	GA	GAll
start	1990-01-02	1990-01-02	1990-01-02	1972-02-01	1972-02-01	1983-01-03	1972-02-01	1979-01-01	1978-01-02	1972-02-01	1972-02-01
end	2013-05-31	2013-05-31	2013-05-31	2013-05-31	2013-05-31	2013-06-03	2013-06-03	2013-06-03	2013-06-03	2013-06-03	2013-05-31
$\mu(\times 10^3)$	0.387	0.187	0.316	0.314	0.124	0.024	0.516	0.238	0.322	0.154	0.159
	(5.3)	(3.0)	(3.4)	(6.2)	(6.7)	(0.6)	(5.0)	(4.2)	(5.3)	(5.8)	(5.6)
$\omega(\times 10^6)$	0.569	0.616	0.364	0.298	0.024	0.027	1.525	0.455	0.619	0.034	0.035
	(2.4)	(4.3)	(3.3)	(4.1)	(3.3)	(1.4)	(4.1)	(4.3)	(3.9)	(1.6)	(1.6)
α	0.089	0.160	0.094	0.104	0.107	0.060	0.055	0.092	0.074	0.037	0.046
	(6.4)	(9.8)	(7.7)	(12.4)	(12.1)	(4.1)	(9.0)	(9.0)	(6.9)	(3.7)	(4.2)
γ	-0.020	-0.007	-0.022	-0.026	-0.024	-0.002	-0.008	-0.020	-0.002	-0.011	-0.007
	(-1.6)	(-1.6)	(-2.0)	(-2.8)	(-2.0)	(-1.6)	(-2.3)	(-1.8)	(-1.6)	(-1.6)	(-0.8)
β	0.912	0.848	0.918	0.907	0.909	0.945	0.940	0.909	0.916	0.966	0.955
	(-5.9)	(-11.4)	(-8.9)	(-11.1)	(-11.4)	(-4.2)	(-8.1)	(-9.6)	(-7.1)	(-2.9)	(-3.2)
	Regression estimates by Asset Class										
$\alpha(\times 10^2)$	0.053	0.083	0.067	0.036	0.016	0.082	0.177	0.144	0.096	0.065	0.034
	(1.3)	(1.8)	(1.4)	(1.5)	(1.5)	(4.7)	(3.0)	(4.4)	(2.8)	(4.0)	(2.8)
$\hat{\sigma}_{126}$	0.334	0.126	0.159	0.227	0.280	0.475	0.161	0.125	0.233	0.113	0.180
	(4.7)	(1.8)	(2.1)	(4.3)	(5.1)	(5.7)	(2.2)	(1.8)	(3.5)	(1.1)	(2.2)
$\hat{\sigma}_{\text{GARCH}}$	0.561	0.754	0.758	0.682	0.632	0.220	0.665	0.581	0.594	0.637	0.655
	(8.0)	(11.4)	(9.9)	(13.1)	(11.6)	(2.9)	(9.1)	(9.3)	(9.2)	(6.1)	(7.9)

D Calculation of Variance Swap Returns

We calculate the returns to a daily variance swap on the S&P 500 using daily observations on the SPX and the VIX, and daily levels of the one-month Treasury bill rate. The historical daily observations on the SPX and the VIX, beginning on January 2, 1990, are taken from the CBOE's VIX website.[20] The daily one-month interest rate series is taken from Ken French's data library.

The VIX is calculated using a panel of S&P 500 index options with a wide range of strike prices and with two maturity dates – generally the two closest-to-maturity contracts, weighted in such a way so as to most closely approximate the swap rate for a variance swap with a constant maturity of 30 calendar days.[21] The calculation method used by the CBOE makes the VIX equivalent to the swap rate for a variance swap on the S&P 500 over the coming 30 calendar days. However, the methodology used by the CBOE is to: (1) annualize this variance; (2) take the square-root of the variance (to convert to volatility), multiply by 100 to convert to percentage terms.

Given the VIX construction methodology, we can calculate the daily return on a variance swap, from day $t-1$ to day t, as:

$$R_{vs,t} = D_t \left[\frac{1}{21} \left(252 \left[100 \cdot \log \left(\frac{S_t}{S_{t-1}} \right) \right]^2 - \mathrm{VIX}_{t-1}^2 \right) + \frac{20}{21} \left(\mathrm{VIX}_t^2 - \mathrm{VIX}_{t-1}^2 \right) \right].$$

Here D_t is the 20-trading day discount factor. This is calculated as $D_t = (1 + r_{1m,t})^{20/252}$, where $r_{1m,t}$ is the annualized one-month treasury bill yield as of day t, from Ken French's website. VIX_t is the level of the VIX as quoted at the end of day t and S_t is the level of the S&P 500, adjusted for all corporate actions, at the end of day t. Note that the factors of 252 and 100 in the equation are because the VIX is quoted in annualized, percentage terms.

This equation is given a flat forward variance curve. That is, we are implicitly making the assumption that the swap rate on 20 trading-day and 21 trading-day variance swap rates on day t are identical (and equal to VIX_t^2). For the market, this approximation should be fairly accurate.

[20] The daily data for the new VIX are available at `http://www.cboe.com/micro/VIX/historical.aspx`.

[21] See Chicago Board Options Exchange (2003) for a full description of the VIX calculation.

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